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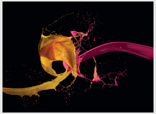

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Is complexity the reason for limited predictability of environmental extremes?

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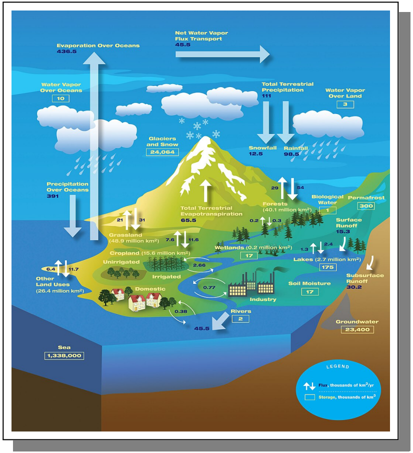
"... although the future is not predictable in any detail, it is manageable as an aggregate phenomenon."
Herbert A. Simon, The Sciences of the Artificial

The Water Cycle: a system with complex dynamics

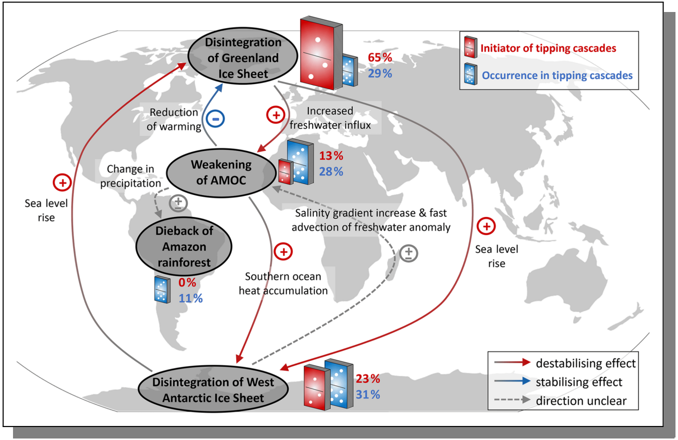


By Atmospheric Infrared Sounder, Public domain, via Wikimedia Commons, NASA/JPL

The water cycle: a complex system

- River flow is determined by several connected systems forming an integrated entity;
- Traditionally these systems are studied and modelled separately. Today we know that to understand patterns we need a holistic approach;
- Processes in a river basin are mostly governed by physical laws that can be expressed in **deterministic form;**
- Models of the water cycle are essentially deterministic.

Complexity in Climate



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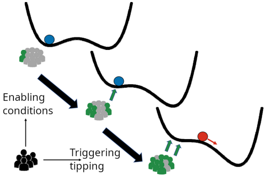
Complexity in the climate systems turns into **tipping points**: critical thresholds that, when crossed, lead to large, accelerating and may be irreversible changes. Tipping points may occur in the hydrosphere and the anthroposphere as well.



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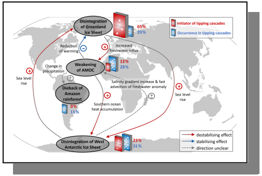
Complexity: definition



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- Physicist Neil Johnson stated that "...even among scientists, there is no unique definition of complexity – and the scientific notion has traditionally been conveyed using particular examples..."
- Complexity arises from non-linear dynamics, sensitivity to initial conditions, chaos;
- Complexity characterizes the behavior of a system or model whose components interact in multiple ways and follow local rules (from Wikipedia);
- **Complexity implies lack of deterministic predictability**.

We will get back to it later →

Complexity: ”my” definition

- **A complex system exhibits patterns structured by deterministic laws enabling limited predictability.**
- The structure of the system cannot be fully compressed without loss, yet is not irreducible randomness.
- It is located somewhere between a fully predictable system (no complexity) and a fully random system (no complexity).
- Typically, the system is predictable up to a finite and limited time horizon.

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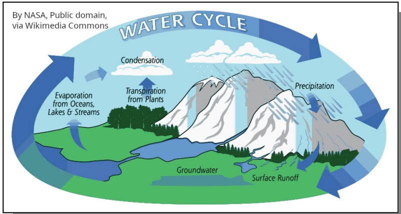
Weather forecasting stone by N Chadwick, CC BY-SA 2.0, via Wikimedia Commons

Reasons for Complexity in Hydrology



Why the water cycle is complex:

- Initial and boundary conditions and forcings cannot be precisely determined;
- Processes are sensitive to initial conditions (butterfly effect) in particular at small scales;
- Unpredictable behaviours in connected systems (social dynamics);
- Physical laws apply at the microscopic scale;
- **Scaling from microscale behaviours to macroscale needs perfect knowledge of the microscale geometry of the system.** →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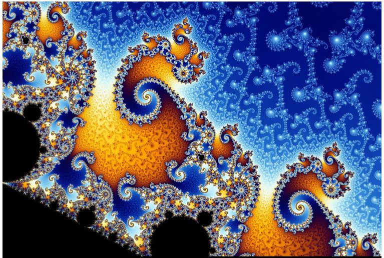
Upscaling microscopic behaviours



The illusion of fractals - One of the major illusions of the new millenium

- Some 40 years ago we dreamed to discover upscaling laws for natural systems;
- A precise upscaling law would allow us to make perfect scaling of environmental systems from microscopic to macroscopic scales;
- Such deterministic description is still to be discovered;

- In abscence of a deterministic relationship to upscale, we resort to a statistical (stochastic) description;
- Macroscale behaviours are computed by estimating their probability distribution. How?

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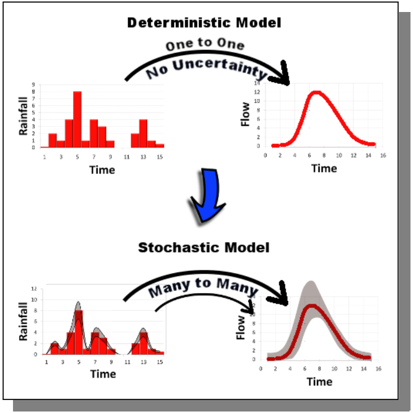
A Phylosophical Consideration

New millenium illusions:

- Endless Progress: The belief that technology and globalization would automatically create peace, prosperity, and equality for all.
- Connection: we felt "more connected than ever," but loneliness, polarization, and misinformation grew.
- Security: Before events like 9/11, financial crashes, pandemics, many believed the world was stable and predictable.
- Infinite Resources: Assuming economic growth and consumption could continue without ecological limits.
- Truth in the Digital Age: With the explosion of information online, many assumed access = accuracy, but it also fueled fake news and disinformation..
- Control: Believing advanced systems (finance, climate management, AI, biotech) are fully under human control, when in reality they often spiral unpredictably.
- Stock market: The Belief that stock market had the capability to self-compensate to ensure equitable distribution of resources and liberalism.

Complexity implies Limited Predictability



Whats does limited predictability means?

- The above reasoning has demonstrated that complexity implies limited **deterministic** predictability;
- However, complexity does not preclude **statistical** predictability;
- In other words, if we cannot make a perfect prediction, we can still make a statistical prediction. How to describe predictions probabilistically?
- This is an open question in hydrology since the 1960s!

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Deterministic Unpredictability is an expected feature of Nature?

The Fortune Teller
Marie-Guillemine Benoist, Public domain, via Wikimedia Commons

What does limited predictability means?

- Unpredictability, rather than precise routine, increases **security of people** against adversaries;
- Social unpredictability increases the resilience against **manipulation**;
- Living organisms are the result of evolution in a competitive environment and therefore they are naturally unpredictable;
- Most organisms evolve a balance: generally predictable enough to cooperate, and unpredictable enough to maximise their security.
- In a similar way, Nature follows laws, which however embed probabilistic and chaotic processes and therefore is intrinsically unpredictable. It is not a weakness, otherwise Nature would not have survived. It is what makes diversity, creativity, and resilience possible.

Getting to Target: Exploiting Statistical Predictabilty

Let's suppose that we deal with a generic deterministic predictive model S for a time series (can be a space time model, the only requirement is that it is deterministic):

$$Q(t) = S\left(\boldsymbol{\Theta}, \mathbf{X}(t)\right) + e(t)$$

where $Q(t)$ is the true value to be predicted, $\boldsymbol{\Theta}$ and $\mathbf{X}$ are model parameter vector and the vector of input variables, respectively, and e is prediction error.

In what follows index t is dropped for clarity.

To exploit statistical predictability we need to transform the above deterministic model into a stochastic model which gives the prediction in terms of probability distribution:

$$f_Q\left(Q\right) = K_S f_{\boldsymbol{\Theta},\mathbf{X}}\left(\boldsymbol{\Theta}, \mathbf{X}\right)$$

How to shape it?

where f indicates a (time varying) probability density function, and K_S is a stochastic operator depending on model S, which operates a shift from one (deterministic) to many (stochastic) predictions.

Stochastic physically-based framework

Getting to Target: Exploiting Statistical Predictabilty

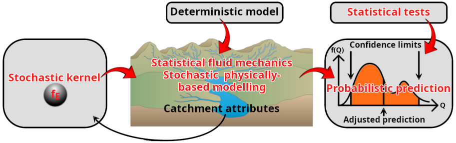
Montanari and Koutsoyiannis (2012) and Montanari (2025, unpublished) proved that the previous equation - under mild assumptions - can be written in the simple form:

$$f_Q(Q) = f_E(Q - S(\boldsymbol{\Theta}, \mathbf{X}))$$

where f_E – in simple words – is the time-varying probability density distribution of the model error.

The key question now becomes the definition of f_E - which depends on model prediction $S(\boldsymbol{\Theta}, \mathbf{X})$ - basing on our **physical knowledge of the system**, by building on the above premises.

What are the hydrological structures, features and patterns determining f_E?



Stochastic kernel f_E: Probability distribution of deterministic model prediction error

Stochastic physically-based framework

Physics Informed Estimation of Stochastic Kernel

Distribution f_E depends on physical behaviours of the system, like for instance catchment attributes, as well as state of the system and prediction model.
Thus, we assume that quantiles of the cumulative distribution function (CDF) F_E of the stochastic kernel f_E, for probability p, are given by two additive components - in quantile function form - namely:

$$F_E^{-1}(p) = F_I^{-1}(p) + F_S^{-1}(p)$$

In the right hand side:

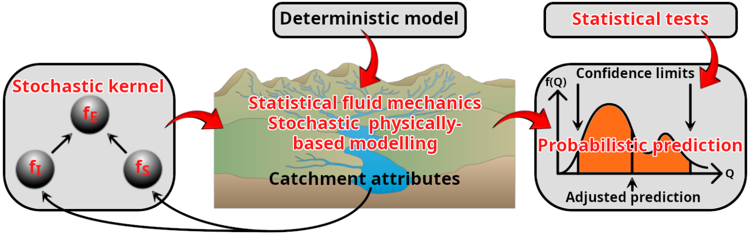
- F_I is the CDF of the prediction error of an ideally optimal model, **which is assumed to be independent of model S**
- F_S is the CDF of an additional error component which depends on model S.

Note:

- the additive property holds if and only if random variables I and S are comonotonic.
- **density f_I is assumed to be a proxy for inherent (irreducible) unpredictability** (Montanari (2025), unpublished).

Physics Informed Estimation of Stochastic Kernel

Resulting picture:



Stochastic kernel f_E: Probability distribution of deterministic model prediction error;
f_I: Probability distribution of inherent predictive uncertainty;
f_S: Probability distribution of model additional uncertainty.

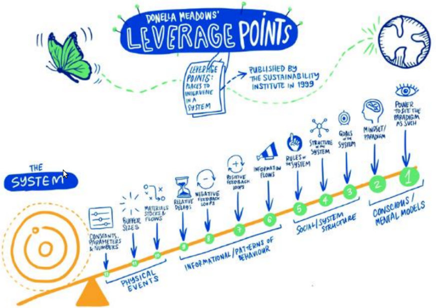
Note:

- Data driven estimation of f_I through machine learning identification of the deterministic part of the input/output transformation.
- Data Driven estimation of f_S through large sample application at the global level.
- Physics informed estimation through analysis of the results of data-driven estimation.

Leverage points

The key role of leverage points to determine system's behaviour

- Pay particular attention to decipher the forcing of leverage points on system behaviours and therefore f_E.
- Leverage points are nodes in a complex system where a "small shift in one thing can produce big changes in everything" (Meadows (2008), Thinking in Systems: A Primer).



Donella Meadows' leverage points (Source: based on Meadows, 1999; credit: UNDP/Carlotta Cataldi)

- Meadows (2008) identified 12 leverage points (see figure), including, for instance, buffer size and material stock and flows.
- Leverage points may amplify a small climate signal generating an environmental disaster. They are one of the causes of "surprise".

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Concluding remarks

Complexity implies lack of deterministic predictability. To manage the system a possible solution is:

- Resort to statistical predictability to maximise preparedness. Failing to take advantage frpm statistical predictability is a missing opportunity for the sake of resilience.
- A rigorous theory is to be adopted to make sure (and test) that statistical predictions are correct in a probabilistic sense. Probability is useful only if it correct and physics informed, otherwise is misleading.
- System thinking and the laws of physics should inform statistical predictability, through the stochastic physically-based estimation of the probability distribution of the prediction error.
- In complex systems, where several subsystems are interacting, and particular when the interaction is with social dynamics, leverage points are to be expected and duly interpreted to maximise (statistical) predictability.