Modeling catchment scale infiltration

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INTRODUCTION

- **OBJECTIVE:** Understanding the processes and the methods for the quantification of water infiltration at the catchment scale.
- **MOTIVATIONS:** Infiltration processes determine the rates and amounts of water available for surface and subsurface runoff, the amounts of water available for evapotranspiration, and the rates and amounts of recharge to ground water. It is crucial to model this process accurately to have good estimates of the different components of the water cycle.

Background

Recall the water mass-balance equation at the catchment scale:

$$\frac{dV(t)}{dt} = P(t) - ET(t) - L(t) - Q(t) \tag{1}$$

where:

- $V(t)$ is the water storage per unit area (defined by the catchment) [L];

- $P(t)$ is the precipitation rate [L/T];

- $ET(t)$ represents evapotranspiration losses [L/T];

- $L(t)$ represents potential losses related to groundwater flux [L/T];

- $Q(t)$ is the surface runoff [L/T].

Assuming that,

(i) the water storage is given by the sum of the groundwater volume, $V_g\ (t)$ and the surface volume $V_s\ (t)$, i.e.:

$$V(t) = V_g\ (t) + V_s\ (t) \tag{2}$$

(ii) during a flood, evapotranspiration and groundwater flow can be neglected, i.e.:

$$L(t) \sim ET\ (t) \sim 0 \tag{3}$$

then, Equation (1) becomes:

$$Q(t) = P(t) - \frac{dV_g(t)}{dt} - \frac{dV_s(t)}{dt} \tag{4}$$

Effective rainfall (i.e., rainfall contributing to runoff) can be defined as:

$$P_e(t) = P(t) - \frac{dV_g(t)}{dt} \tag{5}$$

where:

- $\frac{dV_g(t)}{dt}$ represents infiltration [L/T].

Therefore by substituting Equation (5) info Equation (4) yields

$$Q(t) = P_e(t) - \frac{dV_s(t)}{dt} \tag{6}$$

This lecture will focus on the process of infiltration (Equation (5)), which describes how water (i.e. rainfall) passes across the atmosphere-soil interface and enters a given soil column.

The time-rate at which water infiltrates the soil across the soil interface is known as the **infiltration capacity, $f_c(t)$** [L/T] (Figure 1, solid line). The maximum volume of water which can enter the soil per unit area is known as the **cumulative infiltration, $F(t)$** [L] (Figure 1, dashed line).

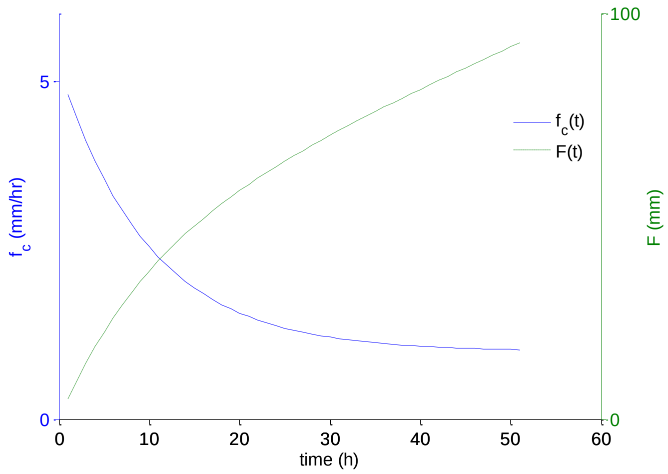

Figure 1: Infiltration capacity and cumulative infiltration

Infiltration capacity is important for engineering studies because it determines how much of the incident rainfall will runoff and how much will enter the soil. Where the input rate exceeds the infiltration capacity of the soil, rainfall excess (otherwise known as effective rainfall) will be generated and contribute to overland flow.

The infiltration process can be described as follows: starting from dry soil conditions, the rainfall tends to infiltrate into the surface soil layers following an asymptotic process, which decreases through time (Figure 1, solid line). Over time, as water redistributes through a soil profile, it displaces air and fills the pores causing more resistance to flow, thereby decreasing the hydraulic gradient and the capacity for infiltration. Indeed, the infiltration capacity at the beginning of the process (unsaturated soil) is maximum, while, as the soil saturates, it decreases towards an asymptotic and constant value. If we consider the cumulative infiltration, $F(t)$, which is the time-integral of $f_c(t)$, it can be easily noticed that $F(t)$ will increase asymptotically towards saturated soil condition (Figure 1, dashed line).

In engineering practice, the infiltration process is most commonly described by two methods: the Horton's and the SCS infiltration methods. Both sets of equations and assumptions will be discussed herein.

Horton's Method:

Horton's method is the earliest infiltration model developed to describe the physical process of infiltration in a quantitative way. It was originally formulated by Robert E. Horton in 1939, and although this approach has been substituted nowadays by more complex and detailed methods, it represents the first serious attempt of describing infiltration processes and quantifying rainfall-excess overland flow for engineering purposes (see *Beven, 2004* for further details on the genesis of Horton's model).

In this model the infiltration process is modeled as an exponential decay of the infiltration capacity in time, from maximum infiltration capacity, f_o, that describes dry, pre-storm conditions, to a minimum, constant infiltration capacity, f_1, (Figure 2). Following the definition given by Brutsaert (pp. 332), the infiltration capacity is most formerly expressed as:

Infiltration capacity

$$f_c(t) = f_1 + (f_o - f_1)e^{-kt} \tag{7}$$

where:

- $f_c(t)$ is the time-dependent infiltration capacity [mm/h];

- f_o is the maximum infiltration capacity [mm/h];

- f_1 is the minimum infiltration capacity [mm/h];

- k is the decay rate of infiltration in time [1/h].

The integral of the time-dependent infiltration capacity, $f_c(t)$, represents the total or cumulative infiltration. We refer to this integrated quantity as **potential cumulative infiltration**, $\boldsymbol{F(t)}$ [mm] (expressed per unit soil surface):

Cumulative infiltration

$$F(t) = \int_0^t f_c(x)\,dx = f_1 t + \frac{(f_o - f_1)}{k}(1 - e^{-kt}) \tag{8}$$

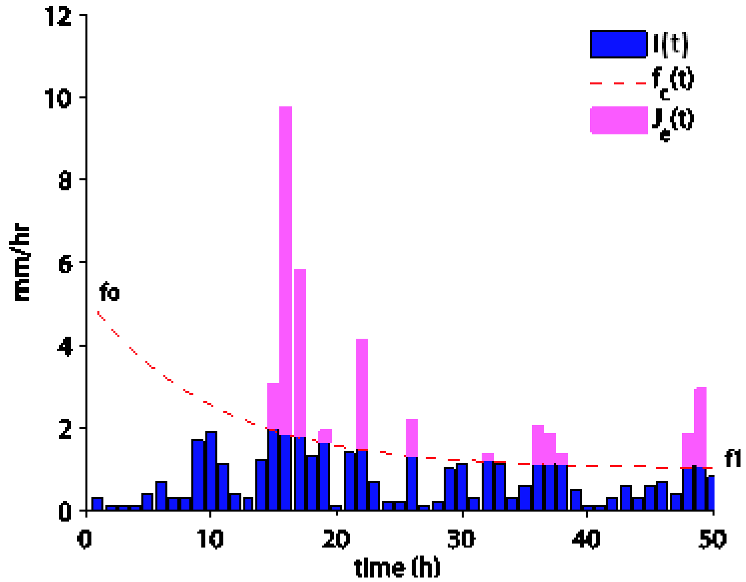

Figure 2: Infiltration rate with initial and maximum infiltration capacities

The form assumed by Equations (7) and (8) is plotted in Figure 1 above, where the solid line represents the infiltration capacity, $f_c(t)$, while the dashed line represents the potential cumulative infiltration, $F(t)$.

As shown in Figure 2, the infiltration capacity curve partitions the rainfall hyetograph (i.e., the rainfall rate $J(t)$ as a function of time) into two parts, allowing the definition of the actual infiltration rate[1], $I(t)$ (the portion of rainfall that infiltrates into the soil, which is represented by the blue portion of the graph below the exponential line) and the rainfall excess $J_e(t)$ (or effective rainfall, i.e. the fraction of the rainfall that flows over the surface and forms the overland flow, which is represented by the pink portion of the graph over the exponential line). The partition of rainfall is described in the following manner:

$$J(t) = J_e(t) + I(t) \tag{9}$$

where:

- $J(t)$ represents the rainfall rate [mm/h];
- $J_e(t)$ is the effective rainfall rate [mm/h];
- $I(t)$ is the actual infiltration rate [mm/h].

[1] The actual infiltration rate, $I(t)$, is smaller than or equal to the infiltration capacity, which represents the maximum rate at which water can infiltrate into soil. See "Actual versus potential cumulative infiltration" for more details.

Because infiltration capacity represents the maximum rate at which water is allowed to infiltrate at a given time, we can define the effective rainfall rate, $J_e(t)$, as the rainfall intensity in excess of the infiltration capacity, for a given time, t:

$$J_e(t) = \begin{cases} 0 & , \quad J(t) \leq f_c(t) \\ J(t) - f_c(t) & , \quad J(t) > f_c(t) \end{cases} \tag{10}$$

The integral counterparts of $J(t)$, $f_c(t)$, $I(t)$, and $J_e(t)$ represent the volume of cumulative rainfall $P(t)$, potential cumulative infiltration $F(t)$, actual cumulative infiltration $F_a(t)$, and cumulative effective rainfall $P_e(t)$ (respectively) per unit area [mm] as shown in Figure 3.

Important concept: Actual versus potential cumulative infiltration

The Horton method also distinguishes between the actual versus potential cumulative infiltration. The actual cumulative infiltration, $F_a(t)$, is given by the cumulative sum of the observed infiltration, $I(t)$, as defined by Equation (9), which considers rainfall as a limiting factor. The actual cumulative infiltration is always less than the potential cumulative infiltration. Indeed, $F(t)$ is simply calculated as the integral of $f_c(t)$ using Equation (8), which is derived without considering rainfall constraints. Therefore, by looking at Figures 2 and 3 and analyzing the first 14 hours of the rainfall event, it can be noticed that:

i) $J(t)$ is less than $f_c(t)$;
ii) the whole rainfall infiltrates;
iii) the cumulative sum of the actual infiltration (i.e., sum of blue bars up to t=14 h) is less than $\int_0^{14\,h} f_c(t)\,dt$.

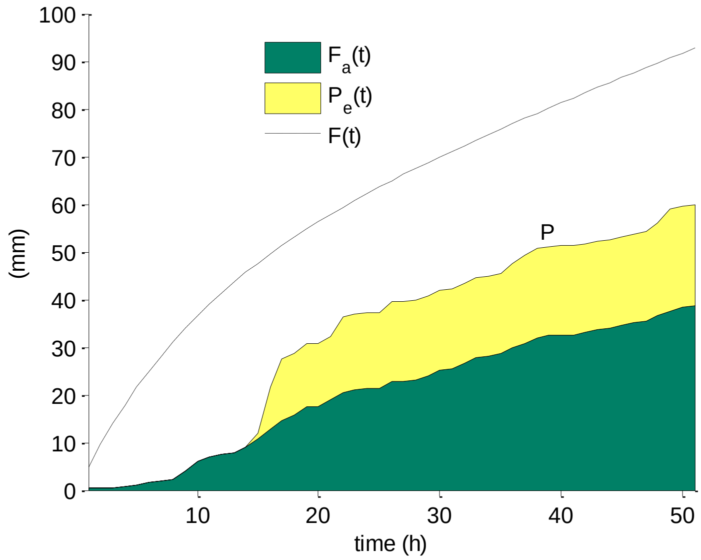

Figure 3: Infiltration components for Horton's method

Finally, the cumulative excess rainfall, $P_e(t)$ (top, yellow shaded region in Figure 3), can be obtained as the difference between the cumulative rainfall and cumulative actual infiltration:

$$P_e(t) = P(t) - F_a(t) \tag{11}$$

SCS Curve Number Method:

The U.S. Department of Agriculture (USDA), now the National Resources Conservation Service (NRCS), formerly known as Soil Conservation Service (SCS), developed an empirical rainfall-runoff relation for small- and medium-sized watersheds. This method developed in 1972 is still appealing and widely used in engineering hydrology due to its simplicity and its good performance compared to other empirical methods. The curve number method is commonly known and referred to as the SCS method. The SCS method relies on an integral representation of the infiltration process. Thus in the following, we will always refer to **cumulative quantities** for:

- rainfall P [mm];

- initial abstraction I_a [mm];

- actual cumulative infiltration F_a [mm];

- rainfall excess P_e [mm].

For the SCS method (Figure 4), three assumptions are maintained:

1) Prior to the beginning of the infiltration process, an initial amount of water is subtracted from the potential rainfall excess (i.e. water constituting the short term surface storage such as puddles or detention ponds). Initial abstraction I_a assumes no rainfall excess, so the potential rainfall excess (runoff) is:

$$P - I_a$$

2) Once the rainfall process begins, an additional depth of water is retained in the watershed. This quantity is the actual cumulative infiltration, F_a, which must be less than or equal to some potential maximum retention S (i.e., maximum water storage):

$$F_a \leq S$$

3) Finally, with enough rainfall, direct runoff develops on the surface. The direct runoff P_e, otherwise known as the the depth of excess precipitation, is always less than or equal to the depth of cumulative precipitation P (that acts as potential rainfall excess):

$$P_e \leq P$$

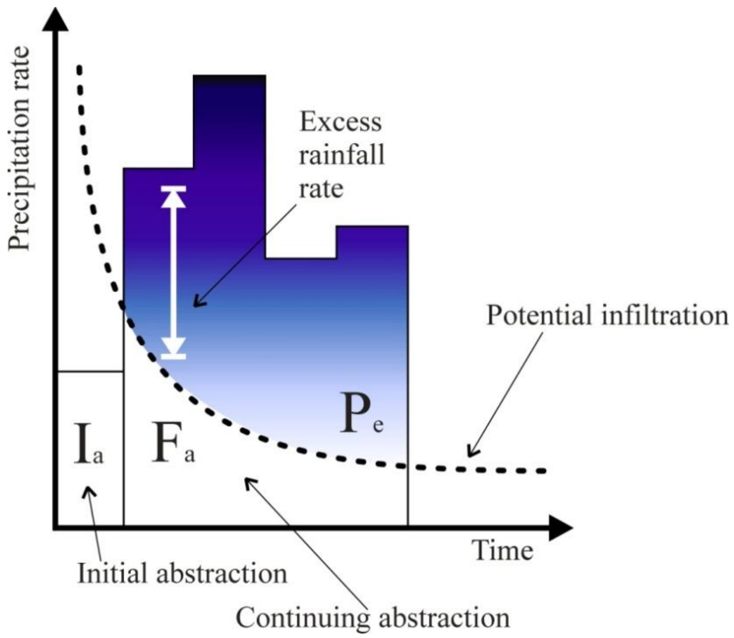

Figure 4: Infiltration components described by the SCS method

The SCS method thereby assumes that the ratio of the actual to the potential cumulative infiltration (i.e., maximum retention) on the left-hand side of Equation (12) is equivalent to the actual to potential rainfall excess ratio:

$$\frac{F_a}{S} = \frac{P_e}{P - I_a} \tag{12}$$

Additionally, it is possible to write the continuity equation for water in the catchment as the sum of all possible contributions which equals the total rainfall:

$$P = P_e + I_a + F_a \tag{13}$$

Solving Equation (13) for $\mathrm{F_a}$ and substituting into Equation (12) gives:

$$\frac{P - P_e - I_a}{S} = \frac{P_e}{P - I_a} \tag{14}$$

This equation can be solved for the rainfall excess, yielding:

$$P_e = \frac{(P - I_a)^2}{P - I_a + S} \tag{15}$$

From empirical studies on numerous catchments, it was found that for normal antecedent moisture conditions the initial abstraction can be obtained as:

$$I_a = 0.2\,S \tag{16}$$

where S [mm] is the maximum (or potential) retention capacity of the catchment that is a function of the land use in the catchment as described in Table 1 below. Note that in any case the initial abstraction cannot exceed the amount of available rainfall in a given time interval.

Values of S are related to the physical characteristics of the watershed through a non-dimensional parameter called the Curve Number (CN), comprised between 0 and 100 that expresses the degree of imperviousness of the surface. A CN = 100 indicates a completely impervious surface for which $P = P_e$, whereas for natural surfaces CN < 100.

Table 1 reports values of CN for different land uses and soil types (expressed by the columns A, B, C, D that go from less to more impermeable soils). The empirical relation between the potential retention, S, and the Curve Number is:

$$S = \frac{25400}{CN} - 254 \tag{17}$$

where S is expressed in mm.

Land Use Description		Hydrologic Soil Group			
		A	B	C	D
Cultivated land: without conservation treatment		72	81	88	99
with conservation treatment		62	71	787	81
Pasture or range land: poor condition[1]		68	79	86	89
good condition[1]		39	61	74	80
Meadow: good condition		30	58	71	78
Wood or forest land: thin stand, poor cover, no mulch		45	66	77	83
good cover[2]		25	55	70	77
Open Spaces, lawns, parks, golf courses, cemeteries, etc.					
good condition: grass cover on 75% or more of the area		39	61	74	80
fair condition: grass cover on 50% to 75% of the area		49	69	79	84
Commercial and business areas (85% impervious)		89	92	94	95
Industrial districts (72% impervious)		81	88	91	93
Residential					
Average lot size	Average % impervious				
1/8 acre or less	65	77	85	90	92
1/4 acre	38	61	75	83	87
1/3 acre	30	57	72	81	87
1/2 acre	25	54	70	80	85
1 acre	20	51	68	79	84
Paved parking lots, roofs, driveways, etc.		98	98	98	98
Streets and roads:					
paved with curbs and storm sewers		98	98	98	98
gravel		76	85	89	91
dirt		72	82	87	89

Table 1: Curve numbers as defined by land use and soil type (from Mays, L.W., *Water Resources Engineering*, Wiley.

References

Brutsaert, W., 2008. *Hydrology. An Introduction.* Cambridge University Press.

Beven, K., 2004. Robert E. Horton's perceptual model of infiltration processes. *Hydrol. Process.,* **18**, 3447–3460.

Mays, L.W., 2005. *Water Resources Engineering.* Wiley.

EXERCISE: Modeling catchment scale infiltration

Given the following rainfall pattern and catchment properties, model the infiltration process and calculate the rainfall excess hyetograph with the SCS method described above.

Rainfall pattern:

Time (h)	Rainfall intensity, j (mm/h)
1	50.8
2	76.2
3	25.4

Watershed properties:

Relative area (%)	Soil use	Soil Group	Curve Number
40%	Residential (1/4 acre-lots)	C	83
25%	Open space, good conditions	D	80
20%	Commercial (85% impervious)	C	94
15%	Industrial (72% impervious)	D	93

Results

Weighted Curve Number: $CN = 0.4 \cdot 83 + 0.25 \cdot 80 + 0.2 \cdot 94 + 0.15 \cdot 93 = 86$

Potential Maximum Retention: $S = \frac{25400}{CN} - 254 = 41.3\text{mm}$

Initial Abstraction: $I_a = 0.2 \cdot \mathrm{S} = 8.3\text{mm}$

	CUMULATIVE QUANTITIES				
Time (h)	**Rainfall, P (mm)**	**Initial Abstraction, I_a (mm)**	**Actual Cumulative Infiltration, F_a (mm)**	**Rainfall Excess, P_e (mm)**	**Rainfall Excess Hyetograph, j_e [mm/h]**
1	50.8	8.3	20.9	21.6	21.6
2	127	8.3	30.6	88.1	66.5
3	152.4	8.3	32.1	112	23.9