Effect of observation errors on the uncertainty of design floods

Giuliano Di Baldassarre\textsuperscript{a,*}, Francesco Laio\textsuperscript{b}, Alberto Montanari\textsuperscript{c}

\textsuperscript{a}Department of Hydroinformatics and Knowledge Management, UNESCO-IHE Institute for Water Education, Delft, The Netherlands
\textsuperscript{b}Dipartimento di Idraulica, Trasporti ed Infrastrutture Civili, Politecnico di Torino, Torino, Italy
\textsuperscript{c}Dipartimento di Ingegneria Civile, Ambientale e dei Materiali, Università degli Studi di Bologna, Bologna, Italy

\textbf{A R T I C L E  I N F O}

Article history:
Available online 8 May 2011

Keywords:
Statistical hydrology
Design flood
Observation uncertainty
Rating curve
Flood risk

\textbf{A B S T R A C T}

This study investigates the uncertainty in the estimation of the design flood induced by errors in flood data. We initially describe and critically discuss the main sources of uncertainty affecting river discharge data, when they are derived using stage-discharge rating curves. Then, different error structures are used to investigate the effects of flood data errors on design flood estimation. Annual maxima values of river discharge observed on the Po River (Italy) at Pontelagoscuro are used as an example. The study demonstrates that observation errors may have a significant impact on the uncertainty of design floods, especially when the rating curve is affected by systematic errors.

\copyright 2011 Elsevier Ltd. All rights reserved.

\textbf{1. Introduction}

A common task in hydrology is the provision of an accurate estimation of the so-called design flood, which can be defined as the discharge value corresponding to a given recurrence interval or return period. The estimation of the design flood is usually carried out by fitting a probability distribution to the observed flood data to suitably represent the frequency of occurrence of rare events (Stedinger et al., 1992). This procedure is affected by uncertainty mainly caused by: (i) difficulties in choosing the appropriate probabilistic model (e.g., Mitosek et al., 2006; Laio et al., 2009); (ii) uncertainty in the parameter values of the model itself (e.g., Martins and Stedinger, 2000; Griffis and Stedinger, 2007); (iii) uncertainty in the river discharge data (considered in this paper). In addition, depending on the specific case study, other sources of uncertainty can be relevant, such as departures from stationarity.

Flood frequency analysis often disregards the uncertainty of river flow data (source iii) used for the statistical inference. However, the scientific literature has shown that errors affecting river discharge observations are indeed far from negligible (Dymond and Christian, 1982; Pelletier, 1987; Kuczera, 1992, 1996; Clarke, 1999; Franchini et al., 1999; Petersen-Øverleir, 2004; Pappenberger et al., 2006; Di Baldassarre and Montanari, 2009; Lang et al., 2010). In fact, river discharge is seldom directly measured during floods, but it is indirectly estimated by measuring the river stage and converting it into river discharge by means of a stage-discharge relationship, namely, the so-called rating curve (Herschy, 1978). This procedure, which is better described in the next section, implies that the uncertainty of river discharge data is particularly significant during flood events, when the rating curve is extrapolated far beyond the measurement range (Rantz et al., 1982; Rosso, 1985; Kuczera, 1996; Pappenberger et al., 2006; Di Baldassarre and Montanari, 2009).

Yet, only few authors have investigated the uncertainty in the design flood caused by imprecision of the river flow data, which is the main objective of this paper. Cong and Xu (1987) analysed this source of uncertainty and concluded that the effects of ordinary measurement errors are slight and essentially negligible. However, Cong and Xu (1987) considered discharge data errors lying in the range 3–5%, which seems to be an optimistic assumption. In fact, many authors showed that the uncertainty affecting flood data might be as large as 30% in many practical cases (e.g. Kuczera, 1996; Pappenberger et al., 2006; Di Baldassarre and Montanari, 2009).

Kuczera (1996) also investigated the effect of errors in river discharge data on flood frequency analysis and proposed an inference framework which makes explicit allowance for observation errors. In particular, Kuczera (1996) showed that errors induced on design flood estimation by the extrapolation of the rating curve can “very substantially, indeed massively” corrupt the estimation of the design flood. However, he concluded that a better characterization of the errors induced by the rating curve is needed to obtain a more precise estimation. Petersen-Øverleir and Reitan (2009) developed a novel methodology for evaluating the joint impact of sample variability and rating curve imprecision on design flood estimation. Yet, they did not take into account the additional uncertainty in river discharge observations induced by extrapolating the rating curve, which is often the main source of error in flood data.
et al. (2010) used a Bayesian framework including a multiplicative error on the rating curve to assess the influence of errors in discharge data on the outcomes of flood frequency analysis.

This study builds on recent improvements in the characterization of the global uncertainty in river discharge observations (Rosso, 1985; Cong and Xu, 1987; Kuczera, 1996; Di Baldassarre and Montanari, 2009) with the aim to investigate the effect of this uncertainty on the estimation of design floods. More specifically, the Kuczera (1996) error structure is used to investigate the effect of rating curve extrapolation error and introduce the rating curve paradox. Secondly, a more comprehensive analysis is performed under the assumption that flow observation errors are Gaussian with zero mean and standard deviation proportional to the true river discharge (Rosso, 1985; Cong and Xu, 1987; Kuczera, 1992). Lastly, a more complex characterization of the global uncertainty in flood data recently proposed by Di Baldassarre and Montanari (2009) is considered.

2. Uncertainty in river discharge observations

In order to better describe the uncertainty that affects river discharge observations, a brief description of the rating curve method for discharge measurement is reported here. The standard methodology to derive a rating curve is based on the assumption that a one-to-one correspondence between the river discharge and the water stage exists (hereafter referred to as the “true rating curve”, see Fig. 1); this assumption is plausible for non-tidal rivers in steady flow conditions (Chow, 1959).

The true rating curve is obviously unknown and the standard procedure to estimate a rating curve consists of carrying out field campaigns to record contemporaneous measurements of water stage, \( h \), and river discharge, \( Q \). Such measurements allow one to identify discrete points \((Q, h)\) that are subsequently interpolated through an analytical relationship that approximates the rating curve (Fig. 1). A power-law function is commonly used in hydrologic practice (Herschy, 1978; Dymond and Christian, 1982),

\[
Q = a(h - b)^c,
\]

where \( a, b, \) and \( c \) are calibration parameters, which are usually estimated by means of the non-linear least squares method (e.g. Petersen-Øverleir, 2004). Eq. (1) is widely used due to its mathematical simplicity and physical plausibility (Chow, 1959; Fenton, 2001; Petersen-Øverleir, 2005).

The uncertainty affecting river discharge observations can be caused by: errors in the individual stage and discharge measurements used to parameterize the rating curve; uncertainty inherent to the least squares estimation of the parameters in Eq. (1); presence of unsteady flow conditions; extrapolation of the rating curve beyond the range of measurements used for its derivation; presence of relevant backwater effects (caused by downstream confluent tributaries, lakes and regulated reservoirs) and temporal changes in the hydraulic properties governing the stage-discharge relationship (e.g. scour and fill, vegetation growth, ice build-up during cold periods).

The error induced by the extrapolation of the rating curve beyond the measurement range (hereafter referred to as “extrapolation error”) is usually the main source of uncertainty in flood data, as recognized by many authors (e.g. Rantz et al., 1982; Kuczera, 1996; Clarke, 1999; Di Baldassarre and Montanari, 2009). This is well known in the literature, as some authors (e.g. Rantz et al., 1982) warn not to extrapolate rating curves beyond a certain range. Nevertheless, many hydrological applications, such as flood frequency analysis, do need river flow data referred to high flow conditions so that the extrapolation of the rating curve beyond the measurement range is often necessary (Pappenberger et al., 2006).

As an example, Fig. 1 shows the effect of rating curve extrapolation beyond the measurement range and highlights the fact that extrapolation errors increase for higher values of the river discharge (see also Reitan and Petersen-Øverleir, 2008). Moreover, Fig. 1 shows an additional interesting aspect: the extrapolation of the rating curve often results in either a systematic underestimation or overestimation. This implies that errors in discharge observation are somehow correlated (Kuczera, 1996).

3. Simplified approach for the qualitative assessment of design flood uncertainty induced by data errors

This section aims at showing, with a simplified approach, the practical effects introduced by the extrapolation error of the rating curve. This simplified approach is useful for obtaining a first qualitative assessment. A more refined analytical derivation will follow in Section 4.

In order to evaluate the impact of the extrapolation error on design flood estimation, annual maxima values of river discharge observed for Po River (Italy) at Pontelagoscuro from 1920 to 1991 are used as an example. This reach of the Po River can be considered

Fig. 1. Example of how errors induced by the extrapolation of the rating curve often lead to either a systematic underestimation [left panel] or overestimation [right panel].
representative of many non-tidal alluvial rivers (Di Baldassarre and Claps, 2011). Moreover, the error structure is derived by referring to the findings of Kuczera (1996), where the extrapolation of the rating curve results in either a systematic underestimation or overestimation (Fig. 1). Kuczera (1996) pointed out that this systematic error can be reliably characterized by assuming that: (a) discharge estimates in the extrapolation domain are corrupted by a relative error; (b) this error depends on the distance from the anchor point (which divides the interpolation from extrapolation zones, e.g. Fig. 1) and not from the origin; (c) interpolation error from the origin to the anchor point is assumed to be negligible.

Thus, according to Kuczera (1996) the systematic error introduced by the extrapolation of the rating curve can be described as:

\[ Q = Q' \quad \text{if} \quad Q' < Q_a (\text{interpolation zone}) \quad (2a) \]

\[ Q = Q_a + \alpha(Q' - Q_a) \quad \text{if} \quad Q' > Q_a (\text{extrapolation zone}) \quad (2b) \]

where \( Q' \) indicates the true value of river discharge, \( Q \) the observed value, \( Q_a \) is the river discharge value corresponding to the anchor point, and \( \alpha \) is a positive-valued coefficient. When \( \alpha < 1 \) extrapolation of the rating curve produces discharge overestimation, when \( \alpha < 1 \) it induces underestimation.

Given that Eq. (2b) is a linear, monotonically-increasing, function, it is easy to derive the relation between the cumulative distribution function (CDF) of the observed value, \( Q \), and the CDF of the true value, \( Q' \) (Benjamin and Cornell, 1970); for large discharge values (which are of interest in flood frequency analysis) one falls in the extrapolation branch of the rating curve, which implies

\[ F_Q(q) = Pr(Q \leq q) = Pr(Q_a + \alpha(Q' - Q_a) \leq q). \quad (3) \]

It follows that:

\[ Q_T = Q_a + \alpha(Q'_T - Q_a), \quad (4) \]

where \( Q_T \) and \( Q'_T \) indicate the quantile, with return period \( T \), derived from the observed and true values, respectively, of river discharge data.

The error structure proposed by Kuczera (1996) enables an approximate investigation of the effect of data errors on the estimation of the design flood. In particular, a numerical experiment is developed as follows (Fig. 2):

(a) annual maxima values of river discharge observed on the Po River (Italy) at the Pontelagoscuro station from 1920 to 1991 are assumed to be true river discharge data, \( Q \);

(b) using a survey of the cross section carried out in 1999, a Manning-type equation is used to derive a steady flow rating curve, which is assumed to be the true rating curve;

(c) the true discharge data, \( Q \), are used to evaluate the 1-in-200 year flood, which is therefore assumed to be the true design flood, \( Q'_T \);

(d) the estimated design flood, \( Q_T \), is estimated by applying Eq. (4), by using a constant value for \( \alpha \) and \( Q_a = 4500 \text{ m}^3 \text{ s}^{-1} \).

Fig. 2 shows the results obtained with \( \alpha = 0.75 \) (left panel) and \( \alpha = 1.25 \) (right panel) and allows an initial interpretation of the practical effects induced by the extrapolation error; these \( \alpha \) values are plausible for the downstream reach of the Po River (Di Baldassarre and Montanari, 2009). One can see that the extrapolation leads to significant uncertainty in the estimation of 1-in-200 year flood.

Hence, Fig. 2 shows the true rating curve (dashed line) and the estimated rating curve (continuous line) and can be used to describe what we call the “rating curve paradox”. To this end, let us assume the (unrealistic) existence of a perfect hydraulic model, i.e. a model perfectly able to reproduce the true rating curve (Fig. 2). Then, we use the estimated design flood as input of this model to predict water levels (or flood extents) corresponding to a certain return period; doing so, we replicate a typical procedure in hydraulic engineering and flood risk management (Di Baldassarre et al., 2010). The results of this test are shown by the black arrows in the diagrams of Fig. 2.

For instance, by considering the left panel of Fig. 2, one can observe that the use of the estimated design flood as input of a perfect hydraulic model would result in a significant underestimation of the design water level. In fact, the true water level corresponding to the true design flood is about 26.4 m (white triangle), while the estimated one is about 23.9 m. It is clear that an underestimation of the 1-in-200 year water level of about 2.5 m may lead, for instance, to inappropriate design of flood defense structures (e.g., levees). Similar conclusions can be reached by analyzing the right panel of Fig. 2. The “rating curve paradox” is that the use of a perfect hydraulic model actually amplifies the uncertainty induced by the use of an imprecise rating curve. In fact,
one (i) observes river stages with a negligible uncertainty, (ii) transforms the river stage in river discharges with an incorrect relationship ("estimated rating curve", Fig. 2) and (iii) transforms the design flood to design flood levels by using a more accurate approach (which, theoretically speaking, should tend to be the perfect hydraulic model) in an attempt to reduce uncertainty. The paradox is that one actually ends up with an incorrect estimate for the design flood levels also because of the use of an accurate model. In particular, we used here a perfect hydraulic model as an extreme example to show not only that it does not compensate for the errors of the estimated rating curve, but also that it potentially amplifies these errors. Thus, when the design flood is reconstructed into water level, the use of the same rating curve (though incorrect) is more appropriate than the application of a hydraulic model (though perfect).

Although the plausibility of the above model for the rating curve error, built after Kuczera (1996), the problem at hand might be much more complex. In fact, while the extrapolation error is often the main source of uncertainty, there are many other uncertainty sources affecting river discharge observations which were not considered here.

4. An analytical approach for estimating design flood uncertainty induced by data errors

This section describes a more refined attempt to analyse the uncertainty in the estimation of the design flood induced by errors in river discharge data. Let $Q'$ indicate the true value of river discharge, $Q$ the observed value and $\varepsilon$ the observation error. It follows that

$$Q = Q' + \varepsilon.$$ (5)

In a first application the observation error, $\varepsilon$, is assumed to be a Gaussian random variable with zero mean and standard deviation proportional to the true river discharge and equal to $\beta Q$ (e.g. Rosso, 1985; Cong and Xu, 1987), where $\beta$ is a positive-valued coefficient. It follows that

$$Q = Q' + \beta Q' \varepsilon',$$ (6)

where $\varepsilon'$ is a Gaussian random variable with mean and standard deviation equal to 0 and 1, respectively. This implies that:

$$\mu_Q = \mu_{Q'};$$

$$\sigma_Q^2 = \sigma_{Q'}^2 + \beta^2 \left( \mu_{Q'}^2 \sigma_{Q'}^2 + \mu_{Q'} \sigma_{Q'}^2 + \sigma_{Q'}^2 \sigma_{Q'}^2 \right)$$

$$= \sigma_{Q'}^2 + \beta^2 \left( \mu_{Q'}^2 + \sigma_{Q'}^2 \right) = \sigma_{Q'}^2 \left( 1 + \beta^2 + \frac{\beta^2}{CV_{Q'}} \right).$$ (8)

It follows that the two parameters of a Gumbel distribution used in the flood frequency analysis, $\theta_1$ and $\theta_2$, can be estimated using the method of moments and read

$$\hat{\theta}_1 = \mu_Q - 0.45 \sigma_Q \sqrt{1 + \beta^2 + \frac{\beta^2}{CV_{Q'}}},$$ (9)

$$\hat{\theta}_2 = \frac{\sqrt{6}}{\pi} \sigma_Q \sqrt{1 + \beta^2 + \frac{\beta^2}{CV_{Q'}}}.\qquad (10)$$

By using the annual maxima values of river discharge observed on the Po River at the Pontelagoscuro as true river flow data, $Q$, for different magnitudes of the error in river discharge observations, $\beta$.

$\varepsilon = \varepsilon_1 + \varepsilon_2,$ (11)

where $\varepsilon_1$ denotes the measurement error of the river flow data that are used to build the rating curve, while $\varepsilon_2$ represents the error induced by incorrect rating curve. Di Baldassarre and Montanari (2009) quantified the $\varepsilon_1$ and $\varepsilon_2$ in detail and assumed that $\varepsilon_2$ can assume positive or negative values with equal probability. In particular, under mild assumptions corresponding to the use of appropriate measurement techniques suggested by European ISO Rule (1997), the following relationship applies (Di Baldassarre and Montanari, 2009):

$$Q = Q' + \varepsilon_1 + \varepsilon_2 = Q' + \gamma_1 Q' \varepsilon_1' + \gamma_2 Q' \varepsilon_2'.$$ (12)

where $\varepsilon_1'$ is Gaussian with zero mean and standard deviation equal to 1, while $\varepsilon_2'$ is a binary variable taking the values +1 or −1 with equal probability ($\gamma_1$ and $\gamma_2$ are positive-valued coefficients). For the reach of the Po River from Isola Sant’Antonio to Pontelagoscuro

Fig. 3 reports the flood quantiles for different return periods, estimated by means of the Gumbel distribution, versus the values of the coefficient $\beta$. One can observe that the larger is $\beta$, the higher is the design flood. This is due to the error structure given by (6), which implies that errors in river flow observations, while not affecting the mean value (Eq. (7)), increase the variance of river discharge data (see Eq. (8)). Consequently, the quantiles increase (Fig. 3). This result was already described in the scientific literature (e.g. Rosso, 1985; Cong and Xu, 1987; Kuczera, 1992). In particular, Fig. 3 shows that for $\beta < 0.05$, the effects of observation errors are negligible, regardless of the considered return period. This also confirms the findings by Cong and Xu (1987). However, as expected, one can see that for increasing values of the error magnitude the effects on quantile estimation are, although conservative for design purposes, no longer negligible. In particular, for $\beta > 0.15$ (which are not unlikely when the rating curve is extrapolated beyond the measurement range; e.g. Pappenberger et al., 2006; Di Baldassarre and Montanari, 2009) the effect of observation errors becomes very relevant.

An additional analysis was carried out by using the findings of Di Baldassarre and Montanari (2009), with the difference that uncertainty induced by the presence of unsteady flow is neglected. Thus, according to Di Baldassarre and Montanari (2009), the global river discharge error is written as

$\varepsilon = \varepsilon_1 + \varepsilon_2,$

where $\varepsilon_1$ denotes the measurement error of the river flow data that are used to build the rating curve, while $\varepsilon_2$ represents the error induced by incorrect rating curve. Di Baldassarre and Montanari (2009) quantified the $\varepsilon_1$ and $\varepsilon_2$ in detail and assumed that $\varepsilon_2$ can assume positive or negative values with equal probability. In particular, under mild assumptions corresponding to the use of appropriate measurement techniques suggested by European ISO Rule (1997), the following relationship applies (Di Baldassarre and Montanari, 2009):

$$Q = Q' + \varepsilon_1 + \varepsilon_2 = Q' + \gamma_1 Q' \varepsilon_1' + \gamma_2 Q' \varepsilon_2'.$$ (12)

where $\varepsilon_1'$ is Gaussian with zero mean and standard deviation equal to 1, while $\varepsilon_2'$ is a binary variable taking the values +1 or −1 with equal probability ($\gamma_1$ and $\gamma_2$ are positive-valued coefficients). For the reach of the Po River from Isola Sant’Antonio to Pontelagoscuro
The error structure can be approximated as:

$$Q = Q' + (\gamma_1 + \gamma_2)Q' \varepsilon^*,$$

(13)

where $\varepsilon^*$ is a generic noise term with zero mean and standard deviation equal to 1. This error structure is similar to the one given by Eq. (6), with $\gamma_1 + \gamma_2 = \beta$. It follows that the parameters of the Gumbel distribution corresponding to the observed river discharge can be estimated by following a similar procedure.

Using this error structure, an exercise was carried out using the annual maxima recorded on the Po River at the station of Pontelagoscuro. Table 1 reports the results of this analysis in terms of estimated quantiles. Hence, this latter application shows that the errors in river discharge data lead to an overestimation of around 10% of the true design flood. Table 1 also reports the flood quantiles estimated in the case when $\varepsilon_2$ would systematically take the value of either $\varepsilon_2 = 0$ or $\varepsilon_2 = 0.027$ in Table 2, without changing in time, which reflects the case where the rating curve is not continuously updated and then systematicity is introduced. In the first and second case (worst case 1 and worst case 2) the rating curve error is producing a systematic underestimation or overestimation, respectively, of about 10 and 20%.

5. Discussion and conclusions

The effects of observation errors on the uncertainty of design floods were analyzed by using different error structures, recently proposed by the scientific literature. The first analysis focused on the extrapolation error (often the main source of uncertainty in flood data) and used Kuczera (1996) error structure. According to this error model a systematic underestimation or overestimation is introduced by the extrapolation of the rating curve (Fig. 1). The effects of this type of error are relevant and may lead to what we called the rating curve paradox, where the imprecision of flood data is amplified by the use of a perfect model to predict water levels or flood extents (Fig. 2). It should be noted that the rating curve paradox can also be seen as an example of transfer of uncertainty from the rating curve, which has low input uncertainty and high structural uncertainty, and the (unrealistic) perfect hydraulic model, which has no structural uncertainty and high input uncertainty.

A second analysis is based on the assumption that observation errors are Gaussian with zero mean and standard deviation proportional to the true river discharge and equal to $\sigma Q$ (Rosso, 1985; Cong and Xu, 1987; Kuczera, 1992). The study showed that the effect of this type of error, although conservative for design purposes, may be relevant for $\beta$ values higher than 0.15 (Fig. 3).

Finally, a third analysis was performed by using the results of Di Baldassarre and Montanari (2009) for the characterization of the global uncertainty in flood data. The results of this study (Table 1) showed that the design flood estimation may be seriously affected by errors in flood data, especially when these are systematic.

The above conclusions point out that reducing the uncertainty of river flow observations is a compelling requirement to improve the reliability of hydrological design variables. One way of reducing this uncertainty is the continuous updating of the rating curve. It is well known that this is a good practice in hydrometry. It is therefore recommended not only to account for changes that may occur in the river geometry and roughness, but also to avoid systematicity, which may substantially increase the uncertainty of design floods. However, one should bear in mind that updating the rating curve is not sufficient to avoid the extrapolation errors and the corresponding large errors in the prediction of flood levels. Another relevant contribution to reducing uncertainty is to collect river flow observations during extreme events, therefore raising the level of the anchor point in the rating curve. This problem is not easy to solve, because to collect river discharge observation during floods is a difficult task. However, the integrated use of innovative remote sensing techniques and hydraulic modeling can provide interesting perspectives for the future (Schumann et al., 2010; Prestininzi et al., 2011).

Acknowledgments

The study has been supported by the Italian Ministry of Education through the Grant 2008KXN4K8 (National Research Project entitled “Uncertainty estimation for precipitation and river discharge data. Effects on water resources planning and flood risk management”).

References


