Nonlinear Dynamics of Extended Hydrologic Systems over long time scales

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Context & Definitions

- Long Time Scale: time over which boundary conditions to the problem change
- Extended Hydrologic Systems: Those that change hydrologic boundary conditions and may themselves be affected by hydrologic state.
- Feedbacks: Negative = Dissipative Dynamics

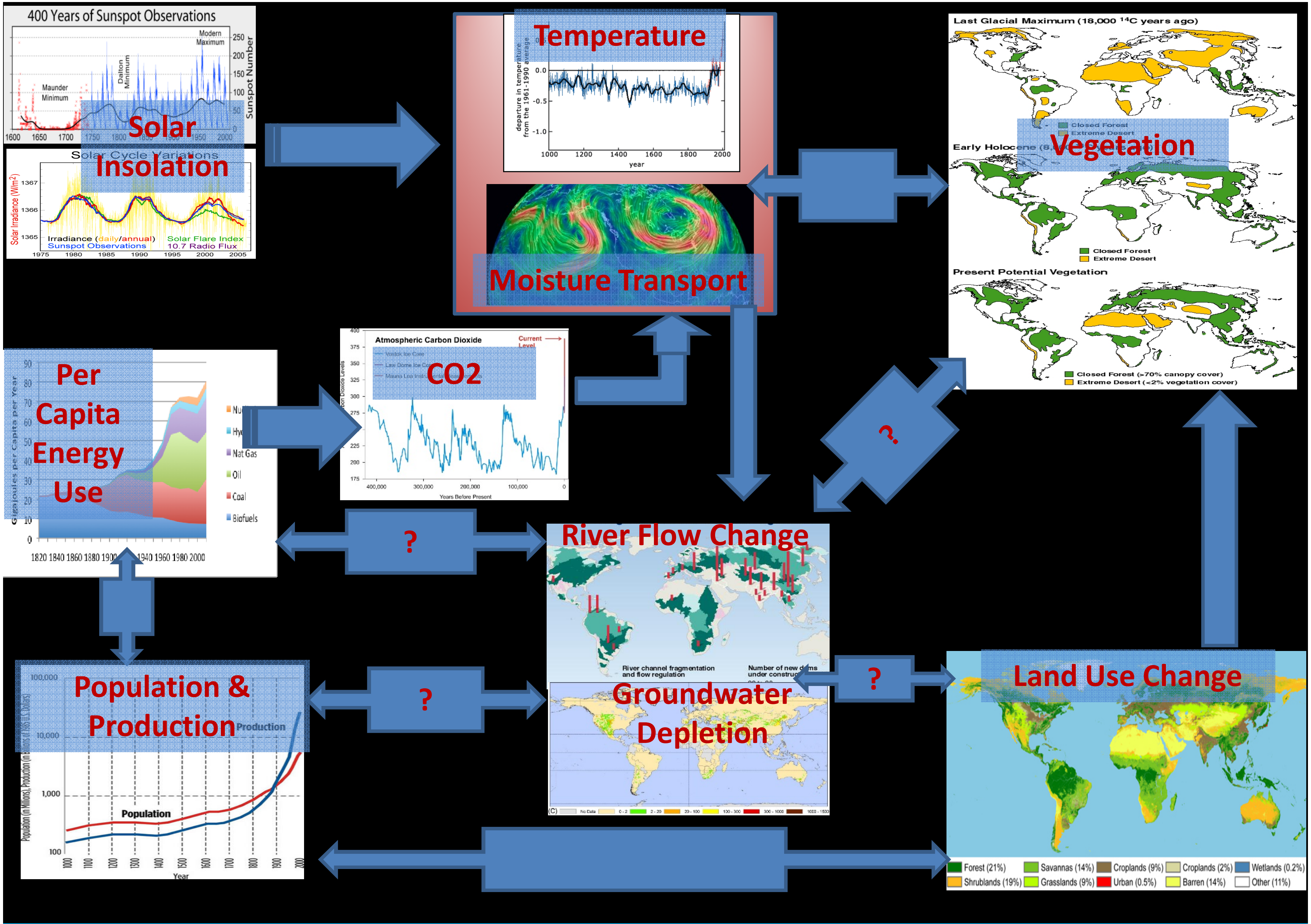
  Positive = Dis-equilibrium Dynamics

  (possible evolution to new regime)

  + and - = Oscillatory Dynamics
- Nonlinearity: Can amplify or dampen, be smooth or threshold
  - instabilities and multi-stable states
- Non-Autonomous Dynamical System $\frac{d\mathbf{x}}{dt} = f(\mathbf{x}, g(\mathbf{y}(t); \boldsymbol{\theta}))$

A Global Scale Example

- Hydrologic System: Spatially distributed physical and chemical state of water in the atmosphere, oceans, ice, vegetation, rivers, reservoirs/lakes, groundwater, and processed water.
- What are the boundary conditions and over what time scales may they be changing?
- What would closure mean for hydrologic dynamics?
  - How can we pose and define useful or interesting problems that could be analyzed?
    - Model complexity – the essential dynamics or a "large model"?



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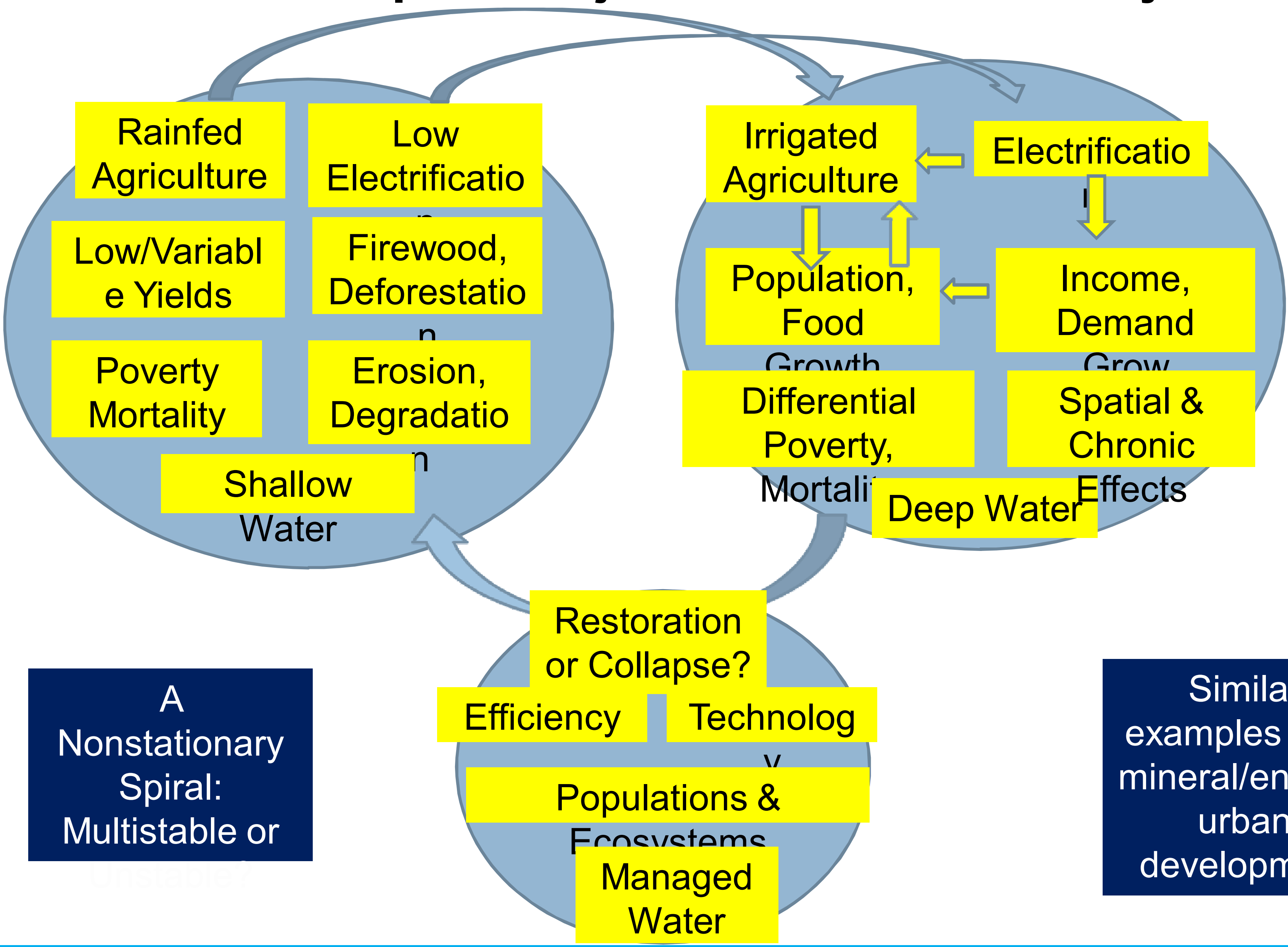
Climate Change?

- Yes! Driven by this question, there has been some progress on this example.
- But, we are still far from having a dynamical model with feedbacks, and still poorly constrained as to:
  - The ability to project or reproduce the hydrologic cycle
  - Characterizing potential (planned or unplanned) adaptation strategies and their feedbacks on to the climate or hydrology
    - Structural relationships between possible changes and hydrologic response and their feedbacks are unclear
    - Adaptation focuses on what to do if scenarios are right, not on what may be long term, and global outcomes if they are implemented
      - We've been geoengineering local hydrology for a long time. Scale?
- What are the relative utilities of large computational models vs small models?
  - Projections of Scenarios vs understanding of tipping points or stability of systems?
  - Role of reconstruction for validating key dynamics?

River Basin Development Trajectories: Natural-Human Dynamics

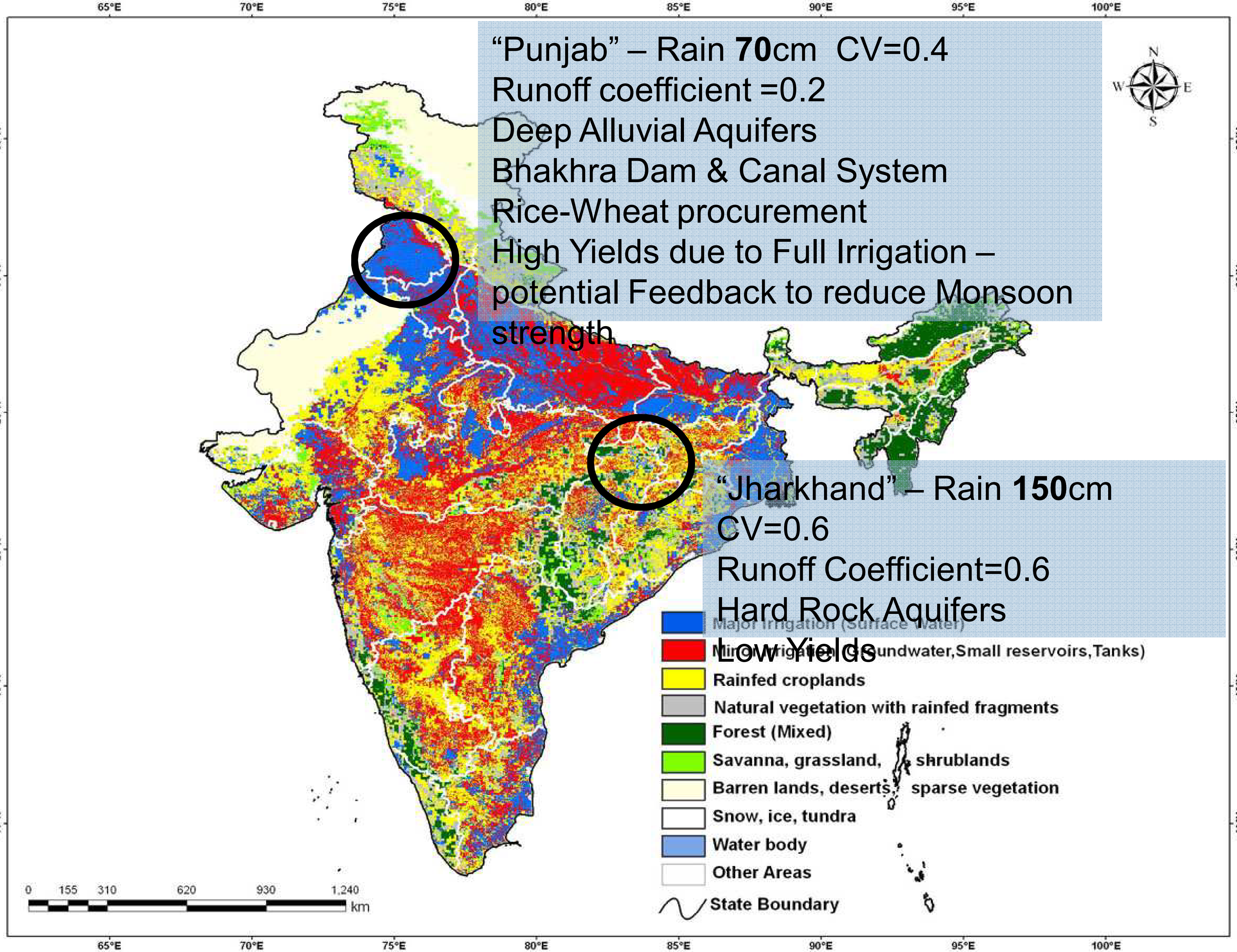


A Nonstationary Spiral: Multistable or Unstable?

Similar examples from mineral/energy/ urban development

Develop this in the Indian Context

- Long Time Scale: 1900-2014
- Extended Hydrologic Systems: Climate, Surface Water, Groundwater, Energy, Crop Choice/Productivity, Economics, Policy (Food Security, Energy Pricing, Procurement Strategy)
- A tale of two regions modeled parametrically considering:
  - Rainfall amount, intermittence and inter-annual variability
  - Surface and Groundwater storage capacity (& development)
  - Price variability as a determinant of regional crop choice
    - Parametric to government procurement strategy
  - Regional crop productivity as a determinant of price variability
  - Energy pricing policy
  - Farmer income and savings
  - Water balance



"Punjab" – Rain **70**cm CV=0.4
Runoff coefficient =0.2
Deep Alluvial Aquifers
Bhakhra Dam & Canal System
Rice-Wheat procurement
High Yields due to Full Irrigation – potential Feedback to reduce Monsoon strength

"Jharkhand" – Rain **150**cm
CV=0.6
Runoff Coefficient=0.6
Hard Rock Aquifers
Low Yields

System Diagram

Time t: Conditions: Storage level, Farmer money supply
Decisions: Crop Area per crop based on last years net income per unit area
Invest in Storage if funds available and there is scope
Nature: Precipitation (including feedback from rice irrigated area, and natural decadal variability)
Outcomes: Storage capacity update
Crop Water Irrigation Requirements per crop
Energy /Diesel used and costs, considering feedback of irrigation needs to electric demand
Storage used and failure amount
Total crop production
Market Price fluctuation of vegetables
net Revenue for the farmer
Change in water Storage level, and Farmer Money supply

A reservoir or aquifer model with input and human behavior responsive to climate

$$\frac{dS}{dt} = I - Q \quad \textit{Conservation of Mass}$$

$$I = LN(\mu, \sigma); CV = \frac{\sigma}{\mu}; \; \mu = \mu_0 + \sum_j A_j \sin(\omega_j t + \varphi_j) \quad \textit{Recharge or Inflow}$$

$$Q = f(S), e.g., = \alpha S \text{ "natural discharge" } \textit{Linear Reservoir}$$

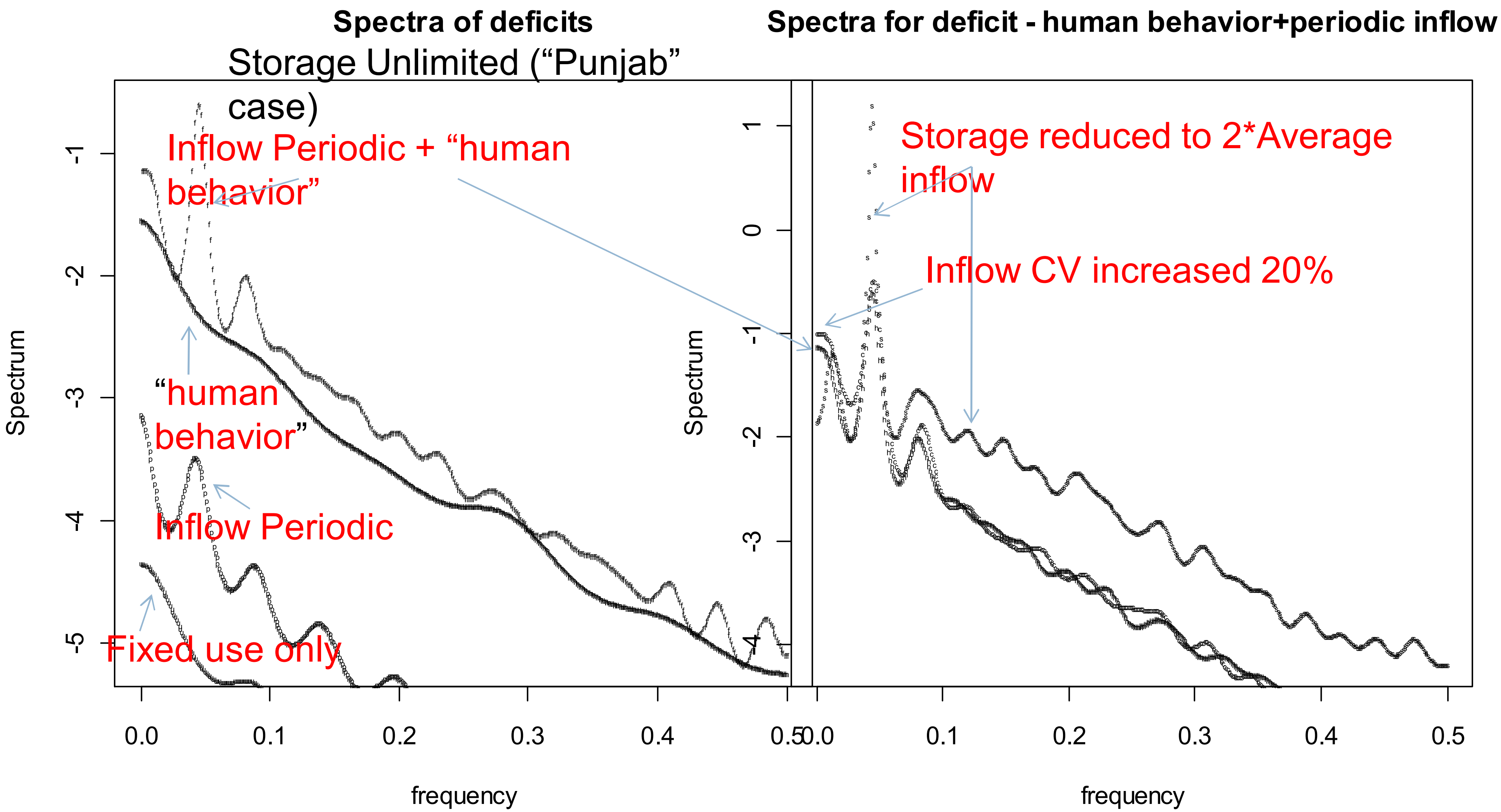
$$+ w_0 \left(1 + \delta(I \le I')\beta_1(I' - I) - \left(1 - \delta(I \le I')\right)\beta_2(I - I')\right) \quad \textit{Human use}$$

$$0 \le S \le S_c \quad \textit{Physical limits on Storage}$$

$$S = \min(S, S_c) \quad \textit{Physical constraint on Storage}$$

$$Q' = \min(Q, S) \quad \textit{Release is limited to storage}$$

$$d = \max(0, Q - Q') \quad \textit{Deficit or shortage}$$



Spectra of deficits
Storage Unlimited ("Punjab" case)
Inflow Periodic + "human behavior"
"human behavior"
Inflow Periodic
Fixed use only
Spectrum
frequency
Spectra for deficit - human behavior+periodic inflow
Storage reduced to 2*Average inflow
Inflow CV increased 20%
Spectrum
frequency

Human Behavior Model (extended)

- Crop Choice (Annual decision):

  A_{rt}=Area allocated to Rice (procurement crop)

  A_{vt}=Area allocated to Vegetables (market crop)

  $A_{rt}+A_{vt} <= A$

P_{rt} = unit price of rice in year t (specified or market)

P_{vt} = average unit price of vegetables in year t = $\beta/f(Yield_{vt} * A_{vt})$ with specified CV

$R_{r(t-1)}, R_{v(t-1)}$ = net revenue per acre from rice & vegetables the previous year (accounts for all costs including water and labor)

$A_{rt} = A_{r(t-1)} + p_r * A_{r(t-1)}$

Where $p_r = (A/A_{r(t-1)} - 1) * Logistic(a(R_{r(t-1)}/R_{v(t-1)}) + b)$

Human Behavior Model

- Aquifer (Punjab) or RWH (Jharkhand) Storage Development
  - Farmer will invest or borrow funds to do so each year
  - For well, only if water level approaches 1m of depth of well
  - Discounting horizon 2 years
  - ΔS_t = Increment of storage capacity added in year t

    = B_t/C_s where B_t= Disposable Savings_t+Borrowing

    such that payback from expected net revenue is 2 years expecting last years prices for each crop

    = $-\delta S_t$, if no funds can be raised to account for degradation

$0<S_t+ \Delta S_t <S_{max}$

  - For Well Pumping, consider reliability of electric supply is ρ_t that depends on the total installed capacity and the

Climate Feedback

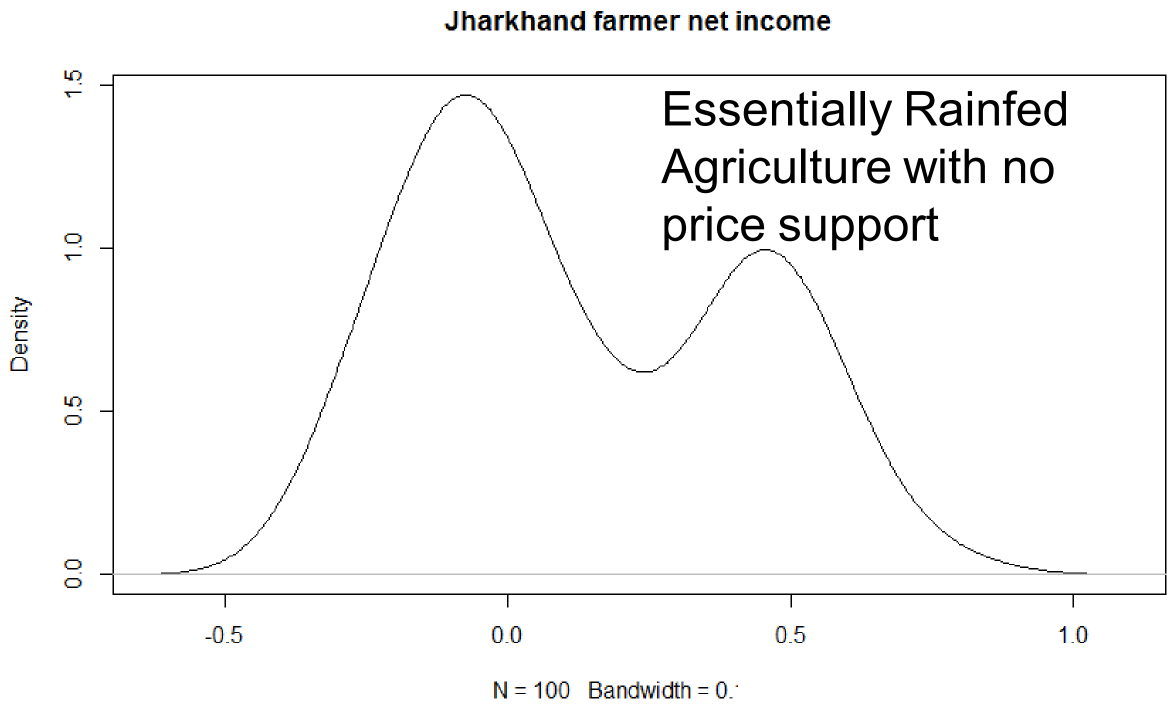
- Indexed to Rice Area in Punjab but not in Jharkhand

$Rain_t \sim LN(\mu_t, CV_{Rain})*g(A_{rt}/A)$

Where $g(A/A_{rt}) = 1-\gamma*Logistic(\theta_1 (A/A_{rt}) + \theta_1)$

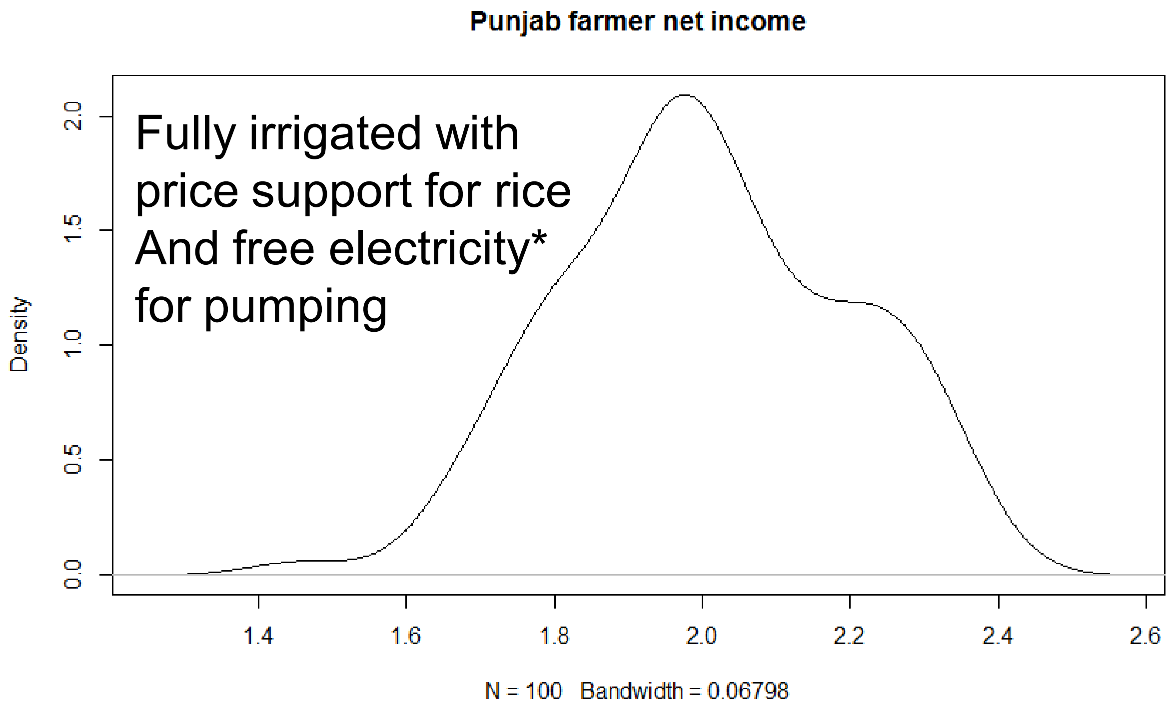
Crop Water requirements from storage

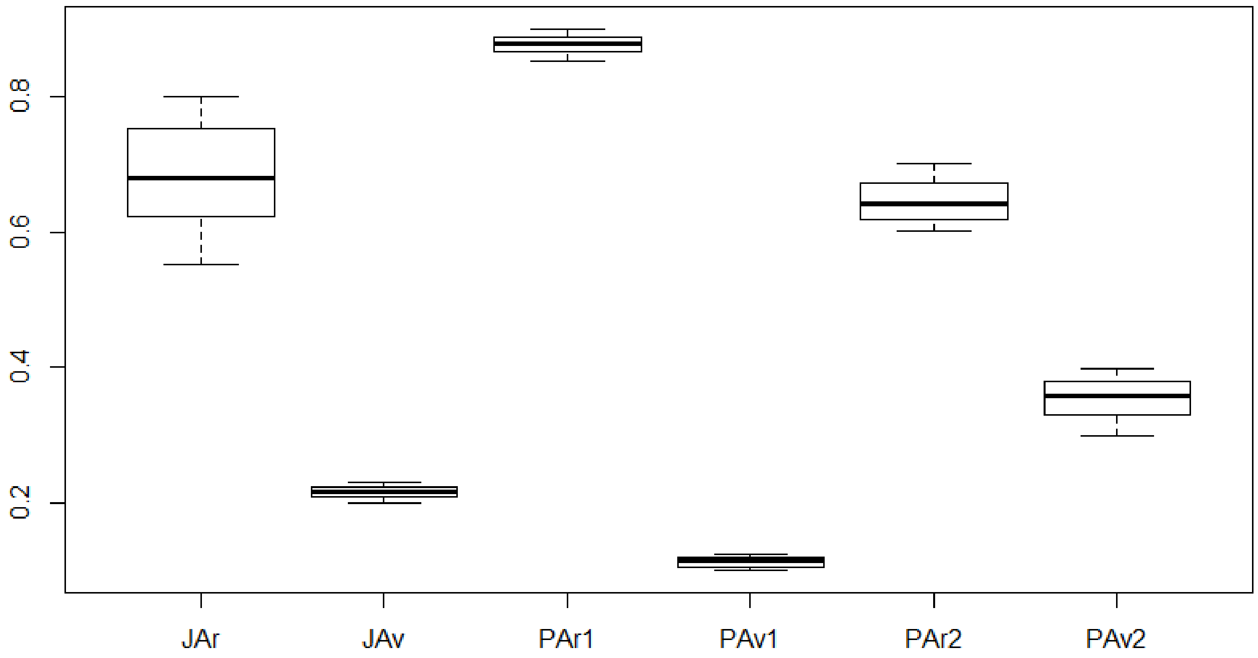
- Based on fixed seasonal cycle of temperature and published ET for each crop (used potato for vegetable) and daily mass balance to compute deficits to be provided from storage or if fails, then reduction in yield proportional to deficit



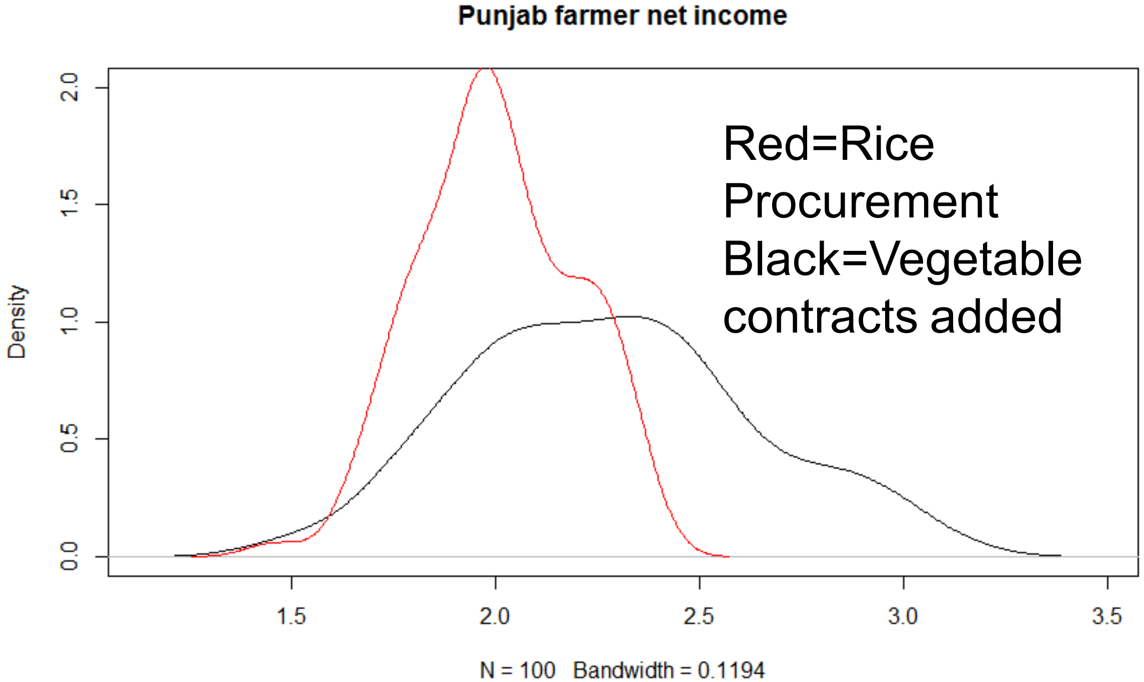
Jharkhand farmer is in a poverty trap- low & highly variable yields and not enough surplus income to invest and sustain storage.

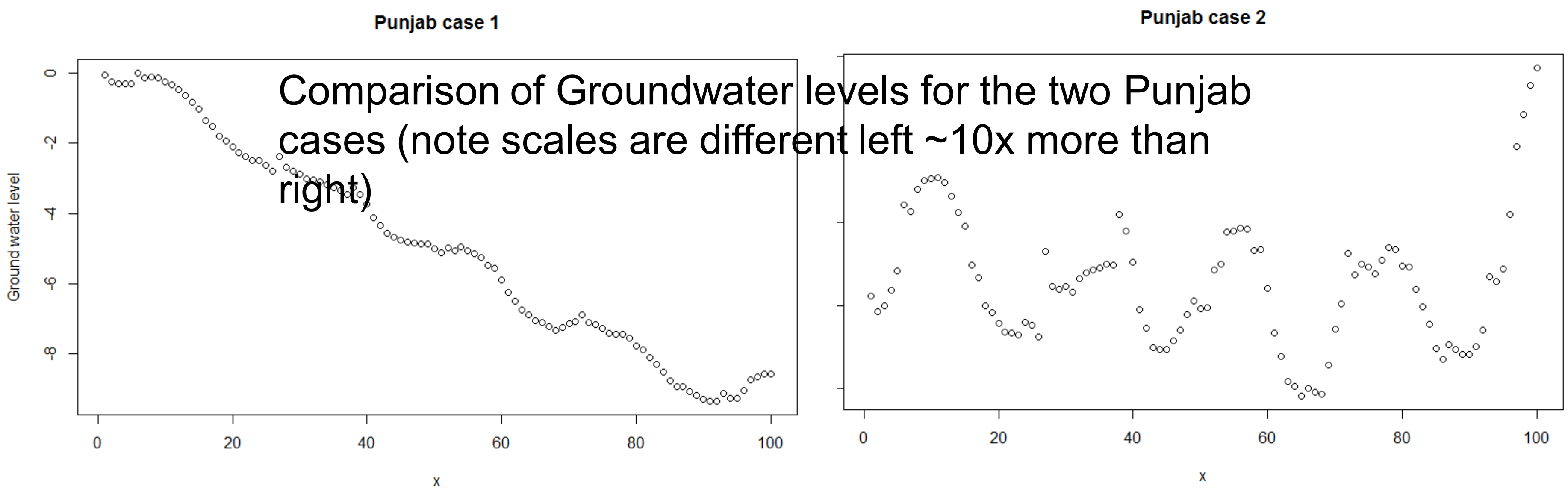
Probability Density plots for 2 conditions
Note scale -0.5 to 1 for Jharkhand
And
1.3 to 2.6 for Punjab





Boxplots showing distribution of Area planted for (Rice, Vegetables)
For
(1) Jharkhand
(2) Punjab with Rice Procured and Market Price fluctuation for Vegetables
(3) Punjab with Rice Procured and reduced market price fluctuation for vegetables (contract farming option)





Comparison of Groundwater levels for the two Punjab cases (note scales are different left ~10x more than right)

Summary

- Panta Rhei raises the interesting question of how we can understand the interaction between hydrology and society. This is very welcome and challenging.
- A dynamical systems view is helpful in framing the questions that arise. One needs to conceptually understand what has changed and why but also what may change, or the model will be necessarily incomplete.
- Spatially distributed computations can be useful for projections, but simpler models that capture the essence of the key features that a thought process (i.e. a hypothesis) frames may also illuminate.
- Prediction of trajectories in the long run may not be possible. However, understanding what may or may not evolve and why provides a bridge between the past and a necessarily uncertain future. Swans can be black and once seen, they can inspire.

India Example

- Jharkhand:
  - Adequate rain but high variability supports rice, but maintains poverty trap and indigenous development of water storage does not emerge, in the absence of government support.
- Punjab
  - Conditional on the policy that the government procures rice at an attractive price and electricity is not priced, rice emerges as the crop of choice since the risk premium for vegetables is not sufficient to drive the choice in that direction.
  - Of the different parameters tried, reducing the CV of the vegetable market price was the most effective in moving the solution to an equilibrium dynamics, with oscillations reflecting those of the climate forcing.