



**European Geosciences Union General Assembly 2014
Vienna, Austria, 27 April – 02 May 2014**

Session HS1.1

Panta Rhei: a vision and an agenda for the next 10 years of hydrological research in support of society

Hydrology, society, change and uncertainty



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Presentation available online: <http://itia.ntua.gr/1441/>

The scientific decade of IAHS 2013-2022

Panta Rhei - Change in Hydrology and Society

View

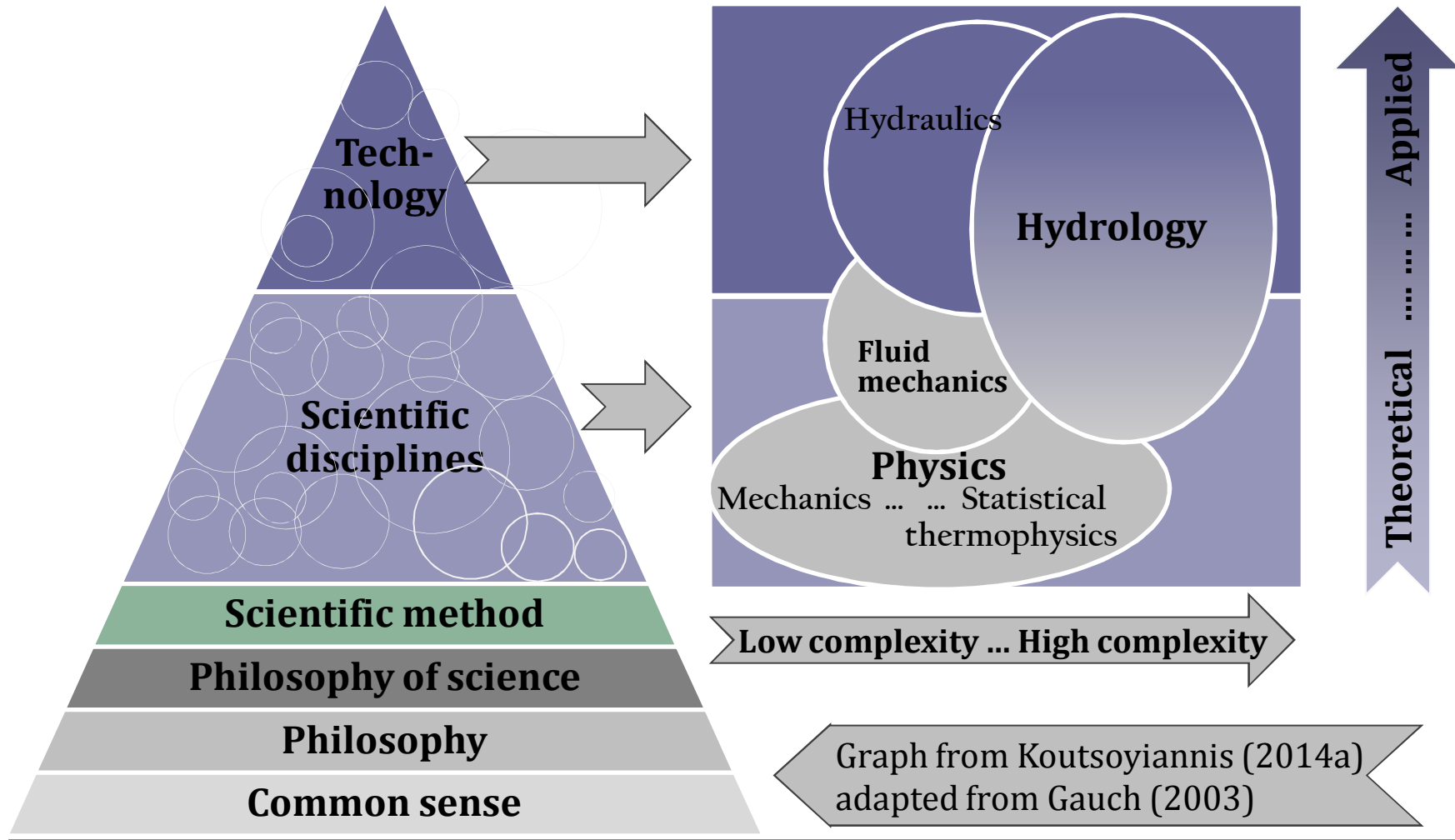
Visitors

The new scientific decade 2013–2022 of IAHS, entitled “Panta Rhei – Everything Flows”, is dedicated to research activities on change in hydrology and society. The purpose of Panta Rhei is to reach an improved interpretation of the processes governing the water cycle by focusing on their changing dynamics in connection with rapidly changing human systems. The practical aim is to improve our capability to make predictions of water resources dynamics to support sustainable societal development in a changing environment. The concept implies a focus on hydrological systems as a changing interface between environment and society, whose dynamics are essential to determine water security, human safety and development, and to set priorities for environmental management. The Scientific Decade 2013–2022 will devise innovative theoretical blueprints for the representation of processes including change and will focus on advanced monitoring and data analysis techniques. Interdisciplinarity will be sought by increased efforts to bridge with the socio–economic sciences and geosciences in general.



<http://distart119.ing.unibo.it/pantarhei/?q=node/1>

The pyramid of knowledge and the roots of hydrology in common sense and philosophy



Heraclitus: Change and randomness

Πάντα ῥεῖ

Everything flows

(Heraclitus; quoted in Plato's *Cratylus*, 339-340)

Ἀἰών παῖς ἔστι παίζων πεσσεύων

Time is a child playing, throwing dice

(Heraclitus; Fragment 52)



Heraclitus
ca. 540-480 BC

Aristotle: Change and nature of precision

Μεταβάλλει τὸ χρόνον πάντα

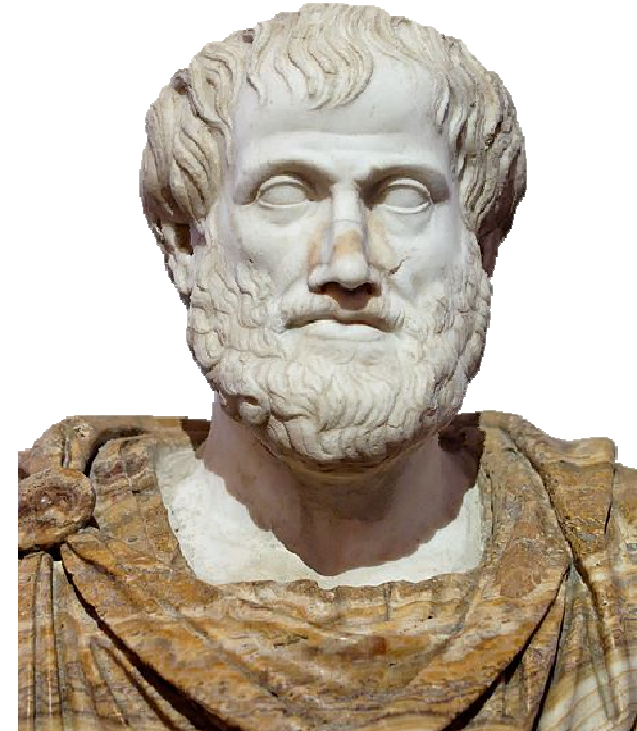
All is changing in the course of time

(Aristotle; Meteorologica, I.14, 353a 16)

Πεπαιδευμένου γὰρ ἔστιν ἕπι πᾶσι τοσοῦτον τὸ κριβεῖν ὅσον ἕπι ζῆτον καθ' ἕκαστον γένος, ὅφρα ἕσον τὸ πράγματος φύσις ἐπιδέχεται

It is the mark of an educated man to look for precision in each class of things just so far as the nature of the subject admits

(Aristotle, Nicomachean Ethics 1094b)

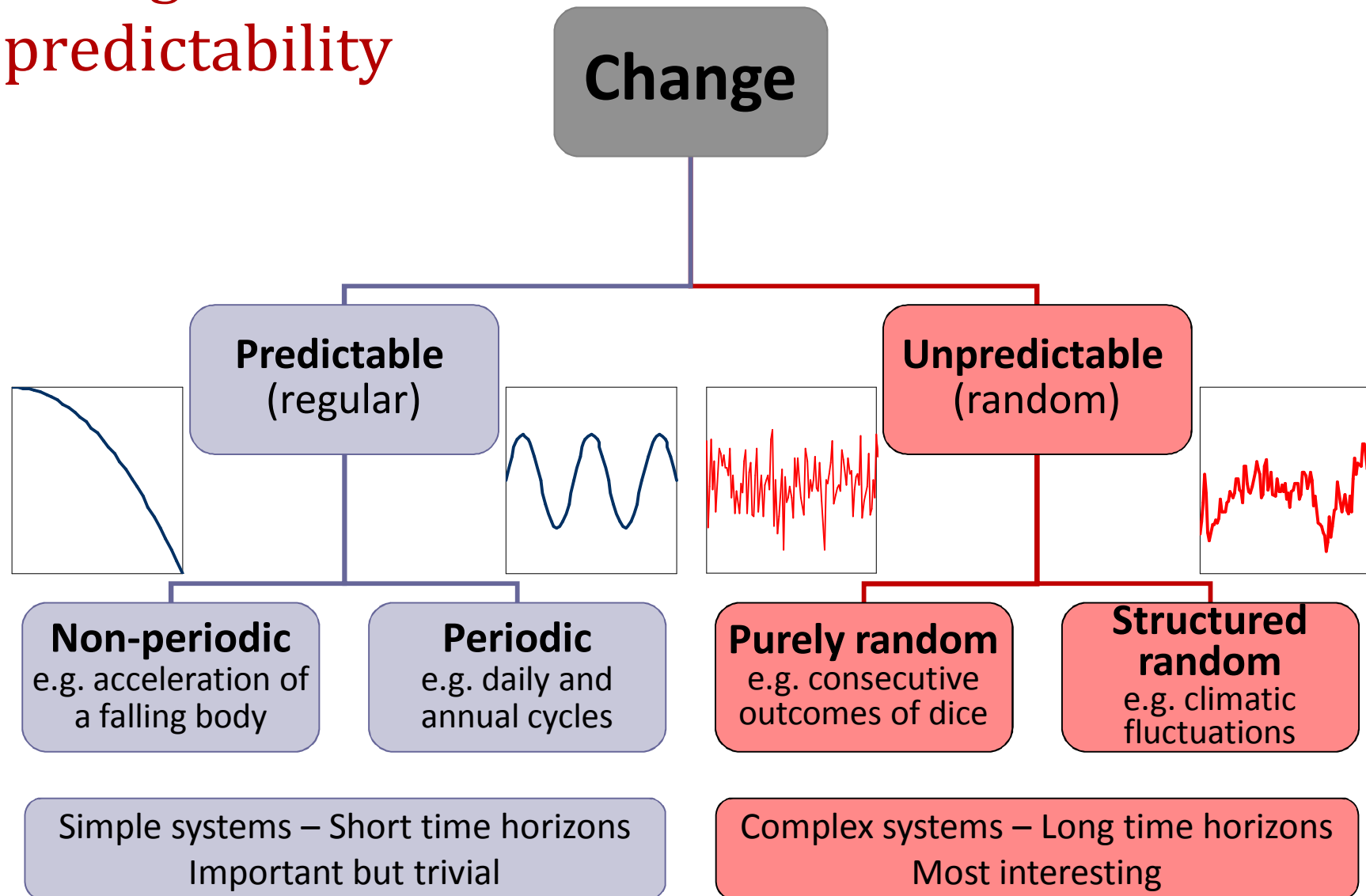


Aristotle

384 – 322 BC

(wikipedia)

Change and predictability



Decision making under uncertainty

Ἰνερρίφθω κύβος *Iacta alea est*

Let the die be cast The die has been cast

[Plutarch's version]

[Suetonius's version]

(Julius Caesar, 49 BC, when crossing Rubicon River)



- ↘ The type of change that can be predicted with precision is usually trivial.
- ↘ Also, decision making under certainty is mostly trivial.
- ↘ History teaches that while understanding and prediction are good advisers for decisions and actions, neither of them is a prerequisite.
- ↘ According to Aristotle, what is needed as a guide to human decisions and actions is **Orthos Logos** (Recta Ratio, or Right Reason).
- ↘ Science, including hydrology, can contribute to societal progress by promoting Orthos Logos.

Social perception of contemporary changes

- ↘ The current acceleration of change, mostly due to unprecedented human achievements in technology, inevitably results in increased uncertainty.
- ↘ In turn, the increased uncertainty makes the society apprehensive about the future, insecure and credulous to a developing **future-telling industry**.
- ↘ Several scientific disciplines, including hydrology, tend to become part of this industry.
- ↘ The social demand for certainties, no matter if these are delusional, is combined by a misconception in the scientific community (cf. Taylor and Ravetz, 2013): **to confuse science with removing uncertainty**.
- ↘ This has been particularly the case in the climate change industry and the part of hydrology related to it.

Future-telling industries: From Delphi and Pythia to modern climate predictions

- Pythia's power relied on ambiguous predictions:
“ὄ-ξεις ὄ-φή-ξεις οὐ θνή-ξεις ἐν πο-λέ-μῳ” or
“you will go you will come not in the war you will die” (put a comma before or after “not”).
- Modern climate predictions (or “projections”) owe their success to the distant time horizon to which they refer (e.g. 2080, 2100, etc.); this makes them (temporarily) resistant to falsifiability.



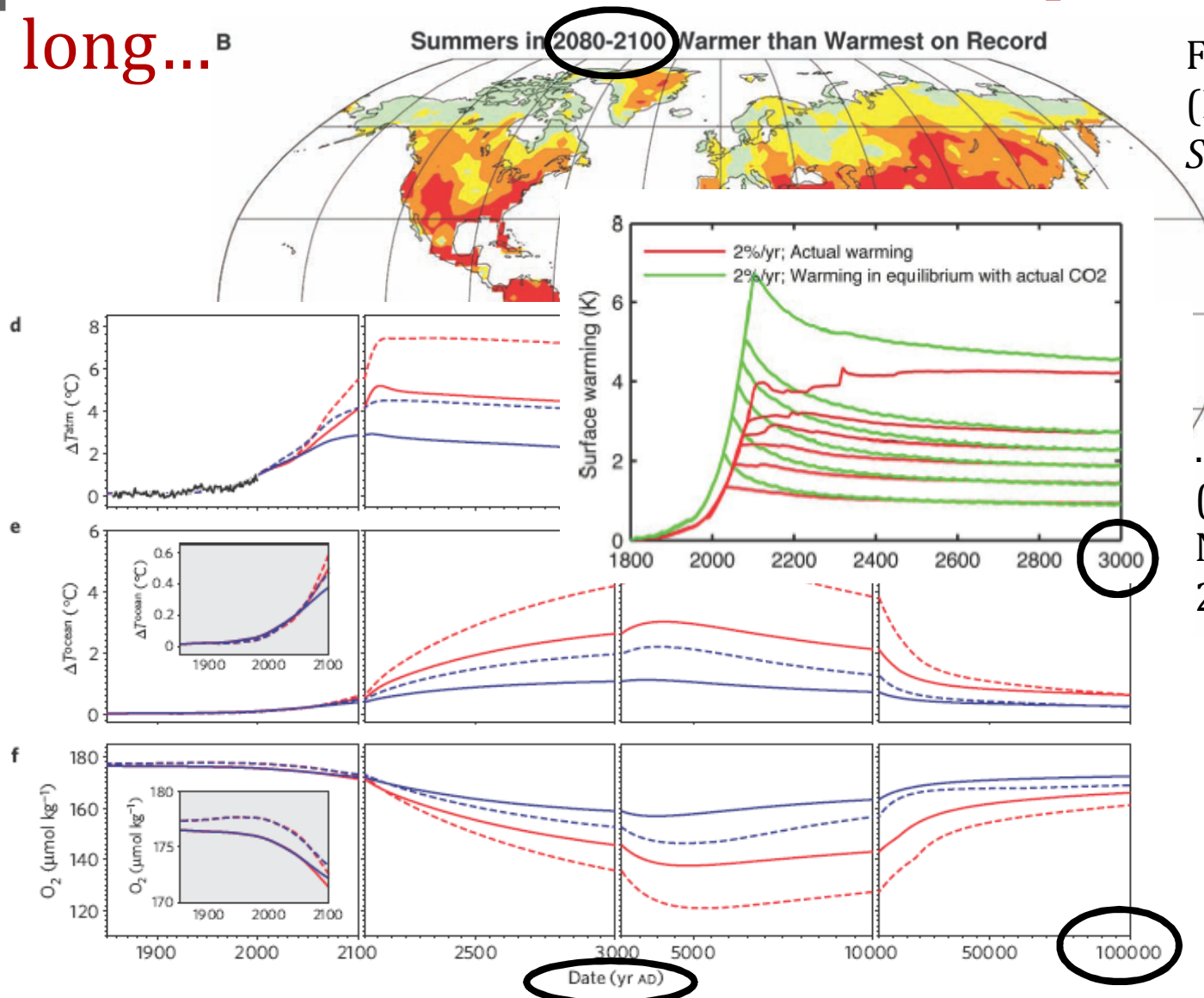
Pythia inspired by *pneuma* rising from below (from wikipedia)

Climate model outputs



(wikipedia)

Indeed, time horizons of climate predictions are long...



From 2100 AD
(Battisti and Naylor,
Science, 2009)...

... to 3000 AD
(Solomon et al.,
Nature Geoscience,
2009)

...to 100 000 AD
(Shaffer et al.,
PNAS, 2009)

How good have climate predictions been so far?

Hydrological Sciences—Journal—des Sciences Hydrologiques

RAPID COMMUNICATION

On the credibility of climate predictions

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Abstract Geographically distributed predictions of future climate, obtained through climate models, are widely used in hydrology and many other disciplines, typically without assessing their reliability. Here we compare the output of various long (over 100 years) records of climatic (30-year) scale. Thus models can perform better at local scale.

See details in Koutsoyiannis et al. (2008, 2011) and Anagnostopoulos et al. (2010).

1334

Hydrological Sciences Journal – Journal des Sciences Hydrologiques, 56(7) 2011

REPLY

Scientific dialogue on climate: is it giving black eyes or opening closed eyes?

Reply to “A black eye for the *Hydrological Sciences Journal*” by D. Huard

D. Koutsoyiannis¹, A. Christofides^{1*}, A. Efstratiadis¹, G. G. Anagnostopoulos² & N. Mamassis¹

A comparison of local and aggregated climate model outputs with observed data

G. G. Anagnostopoulos, D. Koutsoyiannis, A. Christofides, A. Efstratiadis & N. Mamassis

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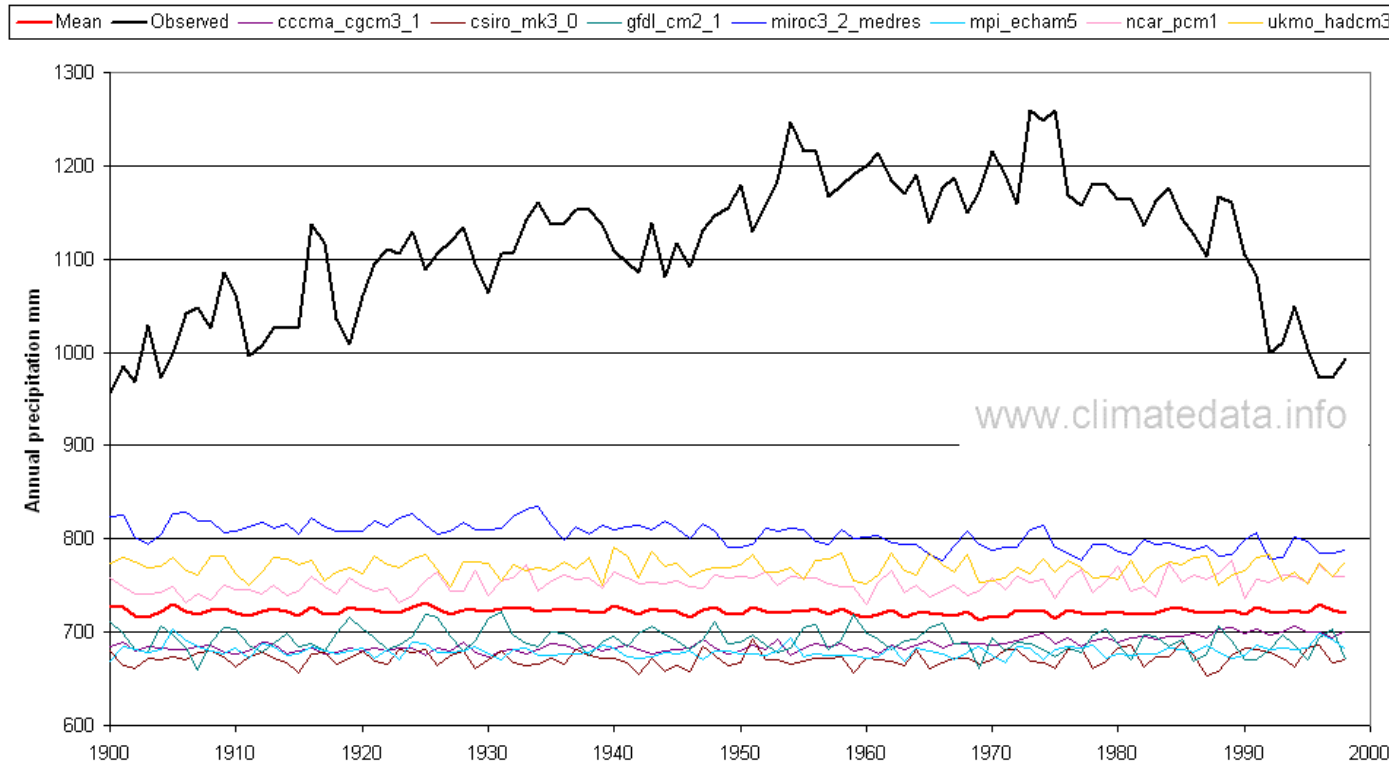
Received 10 April 2009; accepted 10 May 2010; open for discussion until 1 April 2011

Citation Anagnostopoulos, G. G., Koutsoyiannis, D., Christofides, A., Efstratiadis, A. & Mamassis, N. (2010) A comparison of local and aggregated climate model outputs with observed data. *Hydrol. Sci. J.* 55(7), 1094–1110.

Abstract We compare the output of various climate models to temperature and precipitation observations at 55 points around the globe. We also spatially aggregate model output and observations over the contiguous USA using data from 70 stations, and we perform comparison at several temporal scales, including a climatic (30-year) scale. Besides confirming the findings of a previous assessment study that model projections at point scale are poor, results show that the spatially integrated projections are also poor.

Can climate models simulate past precipitation?

Precipitation - alternative simulations - Global - land only



Does the graph indicate:

- ↘ Model biases?
- ↘ Model errors?
- ↘ Model weaknesses?
- ↘ Logico-philosophical problems of the approach followed?

Source: Climate Data Information

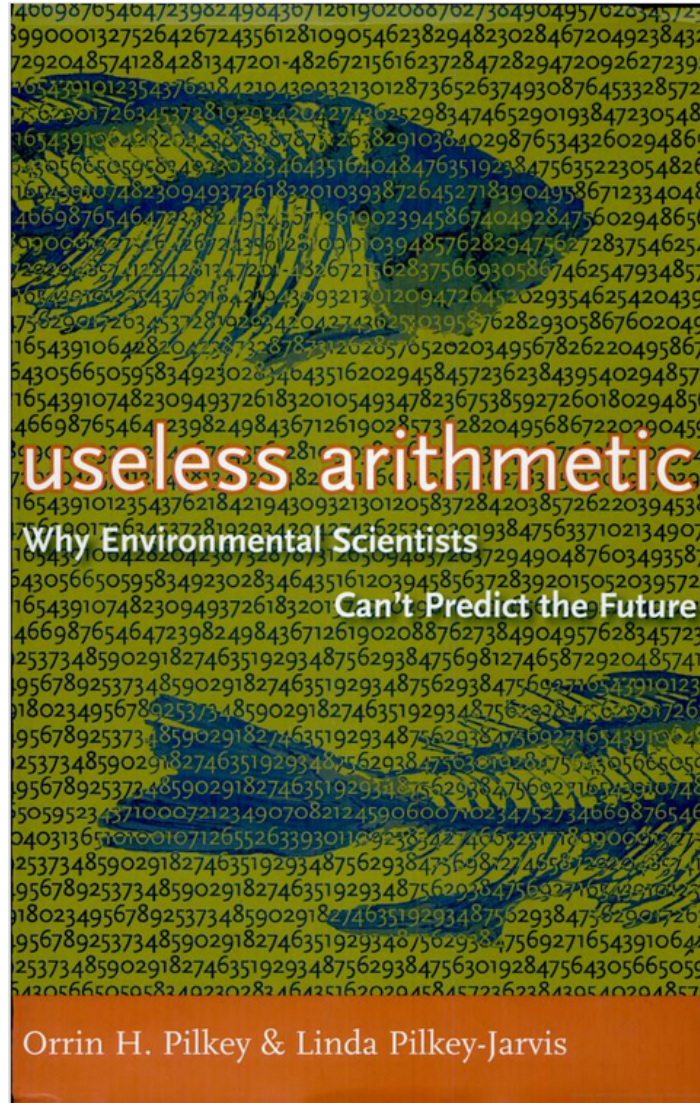
<http://www.climatedata.info/Precipitation/Precipitation/global.html>

Climate prognostology: how useful is it in studying change in hydrology

Very useful: It provides an example that we have just to avoid and it prompts us to search for a different path:

- ↘ Avoid a simplistic view that complex systems can be predictable on the long run in deterministic terms.
- ↘ Avoid being driven by political agendas and economic interests.
- ↘ Avoid mixing up science with activism.
- ↘ Avoid fooling the society by providing unreliable predictions.
- ↘ Avoid promoting biased catastrophic scenarios.
- ↘ Avoid making hydrology “climate impactology”.
- ↘ Exploit the rich experience of hydrology in studying and managing uncertainty.
- ↘ Improve decision making under unpredictability.

Arithmetic simply useless? Or dangerous?



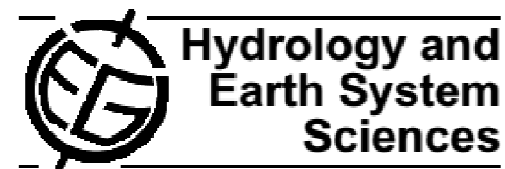
Excerpts from the book:

- ↘ A reviewer of a paper I wrote condemning beach models penned the following criticism, which is very typical of the responses that model critics receive: *“Everyone, even the engineers, realizes that models have shortcomings, some serious ones, but **that is all that they have at this time.** They are constantly working on improving them. Instead of continuing to tear down the existing ones, the discipline would be much better served by offering better alternatives”.*
- ↘ My response (had I been given a chance to respond) would have been this: One should not use bad models for any reason. If you know that there are problems, shame on you and your fellow modelers for not saying so when you apply the model and give the results to the public. **Because of the complexity of beaches, rest assured that nothing better is coming along.** They can never be quantitatively modeled with sufficient accuracy for engineering purposes.

(Pilkey and Pilkey-Jarvis, 2007, p. 136)

A toy model to demonstrate (un)predictability

Hydrol. Earth Syst. Sci., 14, 585–601, 2010
www.hydrol-earth-syst-sci.net/14/585/2010/
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HESS Opinions

“A random walk on water”

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Invited contribution by D. Koutsoyiannis, recipient of the EGU Henry Darcy Medal 2009.

Received: 5 October 2009 – Published in Hydrol. Earth Syst. Sci. Discuss.: 29 October 2009

Revised: 19 March 2010 – Accepted: 19 March 2010 – Published: 25 March 2010

Abstract. According to the traditional notion of randomness and uncertainty, natural phenomena are separated into two mutually exclusive components, random (or stochastic) and deterministic. Within this dichotomous logic, the deterministic part supposedly represents cause-effect relationships and, thus, is physics and science (the “good”), whereas randomness has little relationship with science and no

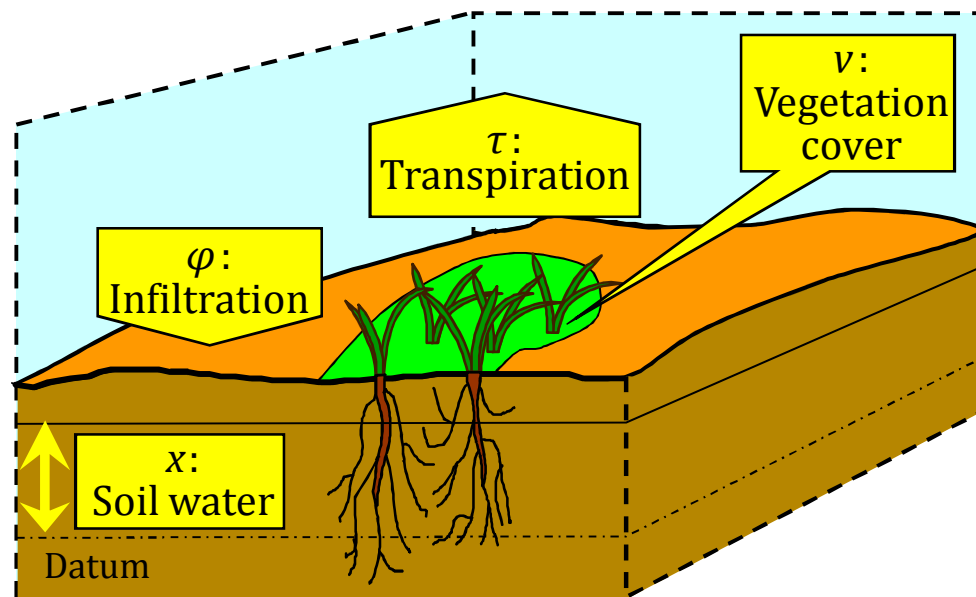
1 What is randomness?

In his foundation of the modern axiomatic theory of probability, A. N. Kolmogorov (1933) avoided defining randomness. He used the notions of random events and random variables in a mathematical sense but without explaining what randomness is. Later, in about 1965, A. N. Kolmogorov and

Emergence of randomness from determinism from a caricature hydrological system

- ↘ The toy model is designed intentionally simple.
- ↘ Only infiltration, transpiration and soil water storage are considered.
- ↘ The rates of infiltration φ and potential transpiration τ_p are **constant**.

Nothing in the model is set to be random.

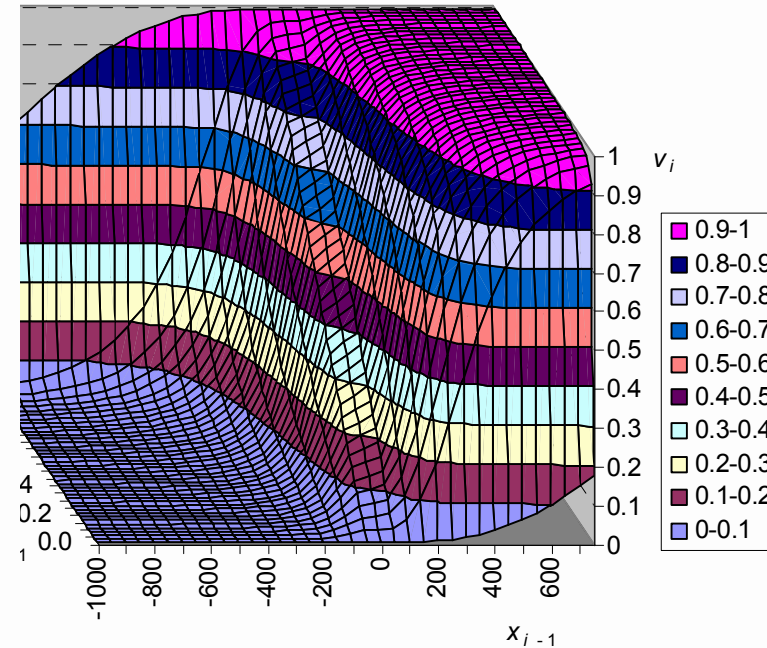
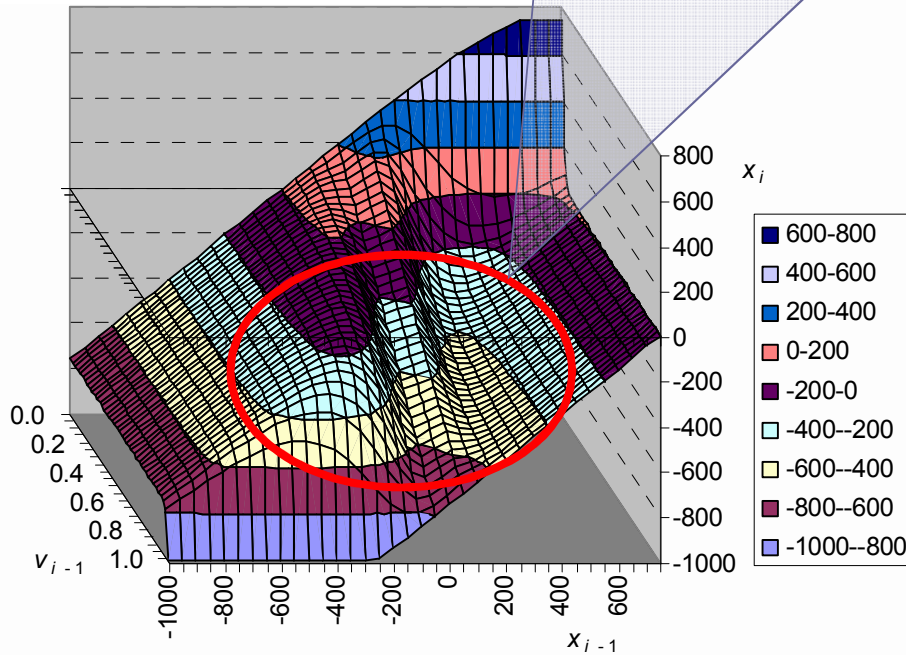


- ↘ Discrete time: i ($t = i\Delta$ where Δ is an arbitrary time unit, $\Delta = 1$ TU).
- ↘ Constants
 - Input: $\varphi = 250$ mm/TU;
 - Potential output: $\tau_p = 1000$ mm/TU.
- ↘ State variables (a **2D dynamical system**):
 - Vegetation cover, v_i ($0 \leq v_i \leq 1$);
 - Soil water (no distinction from groundwater): x_i ($-\infty \leq x_i \leq \alpha = 750$ mm).
- ↘ Actual output: $\tau_i = v_i \tau_p \Delta$
- ↘ Water balance

$$x_i = \min(x_{i-1} + \Delta(\varphi - v_{i-1} \tau_p), \alpha)$$

System dynamics

Interesting surface—Not invertible transformation



water balance - vegetation cover dynamics

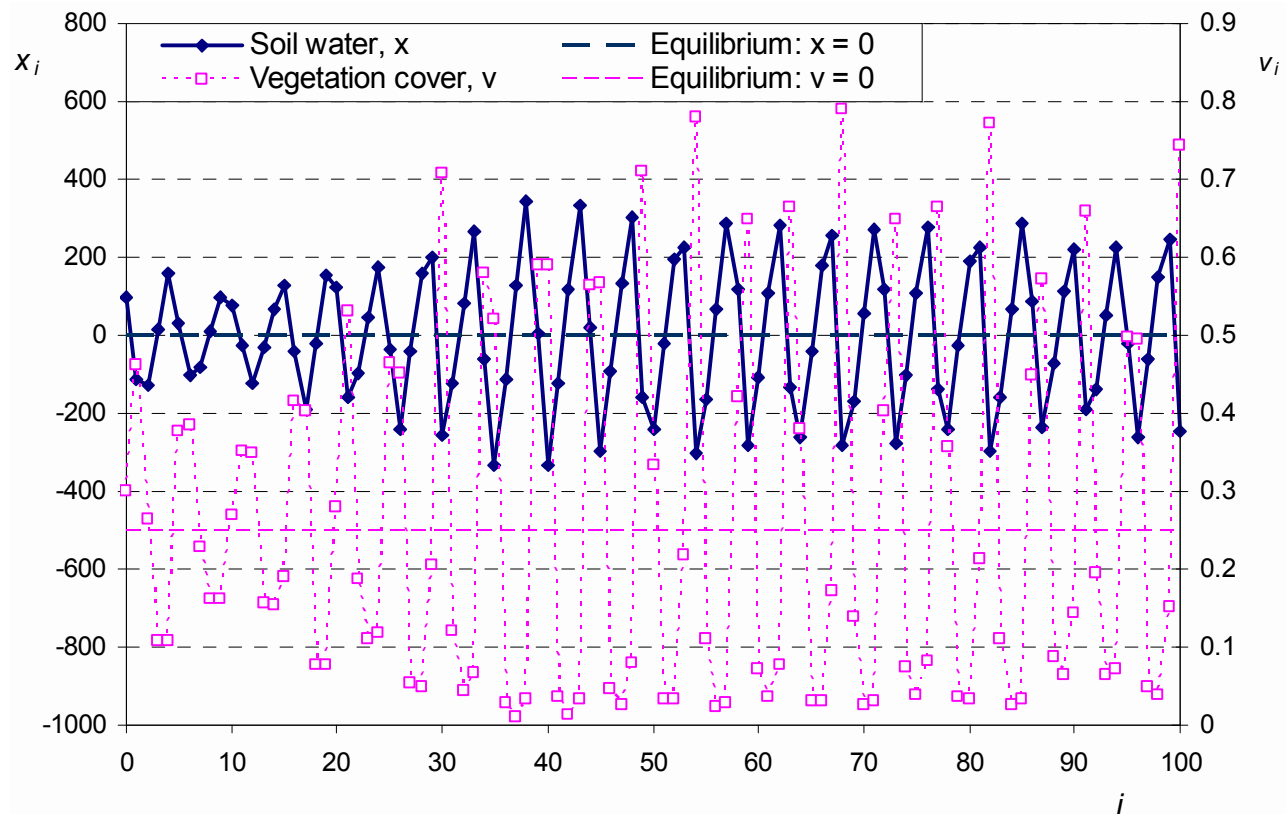
$$x_i = \min(x_{i-1} + \Delta(\varphi - v_{i-1}\tau_p), \alpha)$$

$$v_i = \frac{\max(1 + (x_{i-1} / \beta)^3, 1) v_{i-1}}{\max(1 - (x_{i-1} / \beta)^3, 1) + (x_{i-1} / \beta)^3 v_{i-1}}$$

Assumed constants: $\varphi = 250$ mm/TU, $\tau_p = 1000$ mm/TU, $\alpha = 750$ mm, $\beta = 100$ mm.
Easy to program in a hand calculator or a spreadsheet.

Interesting trajectories produced by simple deterministic dynamics

- ↘ These trajectories of x and v , for time $i = 1$ to 100 were produced assuming initial conditions $x_0 = 100$ mm ($\neq 0$) and $v_0 = 0.30$ ($\neq 0.25$) using a spreadsheet (it can be downloaded from itia.ntua.gr/923/).
- ↘ The system state does not converge to an equilibrium.
- ↘ The trajectories seem periodic.
- ↘ Iterative application of the simple dynamics allows “prediction” for arbitrarily long time horizons (e.g., $x_{100} = -244.55$ mm; $v_{100} = 0.7423$).

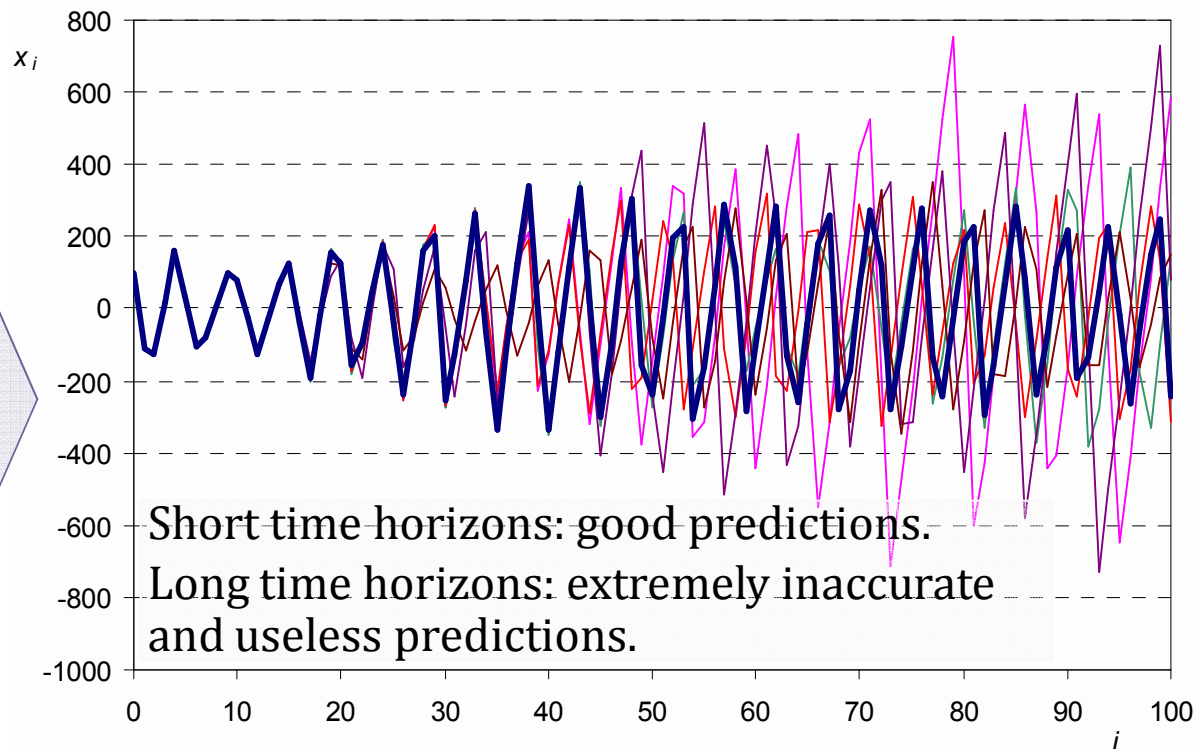


Does deterministic dynamics allow a reliable prediction at an arbitrarily long time horizon?

- ↘ Postulate: A continuous (real) variable that varies in time cannot be ever known with full precision (infinite decimal points).
- ↘ It is reasonable then to assume that there is some small uncertainty in the initial conditions (initial values of state variables).
- ↘ Sensitivity analysis allows to see that a tiny uncertainty in initial conditions may get amplified.

Bold blue line corresponds to initial conditions $x_0 = 100$ mm, $v_0 = 0.30$.

All other lines represent initial conditions slightly (< 1%) different.



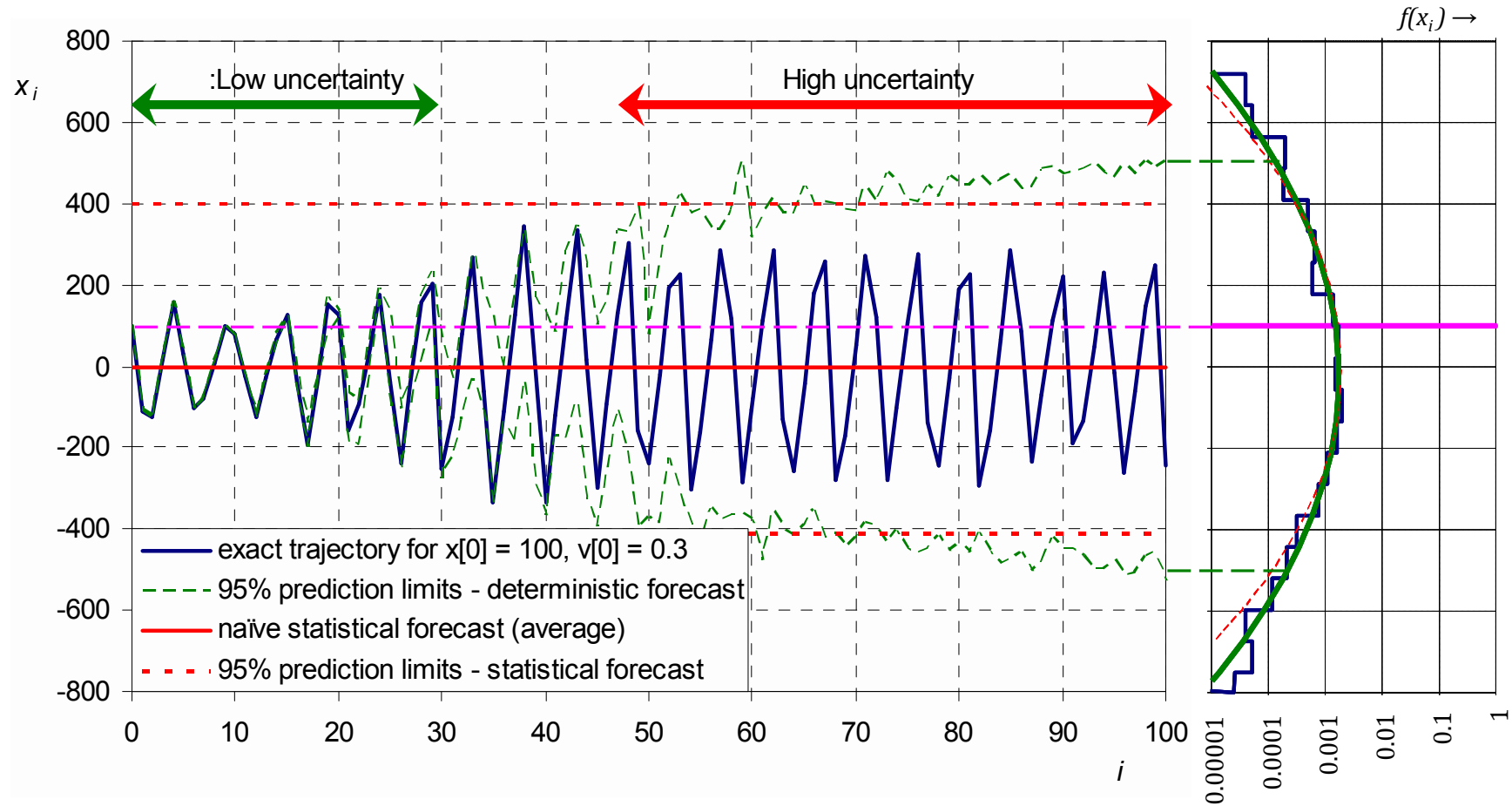
From deterministic to stochastic predictions

- In a deterministic description, $\mathbf{x}_i := (x_i, v_i)$ is the vector of the system state and \mathcal{S} is the vector function representing the known deterministic dynamics of the system.
- Because of the inefficiency of the deterministic description, we turn into a stochastic description and consider $\underline{\mathbf{x}}_i$ as a random variable with a probability density function $f_i(\mathbf{x})$.
- The stochastic representation behaves like a deterministic solution, but refers to the evolution in time of admissible sets and probability density functions, rather than to trajectories of points:

$$\text{From } \mathbf{x}_i = \mathcal{S}(\mathbf{x}_{i-1}) \text{ to } f_i(\mathbf{x}) = \frac{\partial^2}{\partial x \partial v} \int_{\mathcal{S}^{-1}(A)} f_{i-1}(\mathbf{u}) d\mathbf{u}$$

where $A := \{\underline{\mathbf{x}} \leq (x, v)\}$ and $\mathcal{S}^{-1}(A)$ is the counterimage of A .

Stochastics for ever...



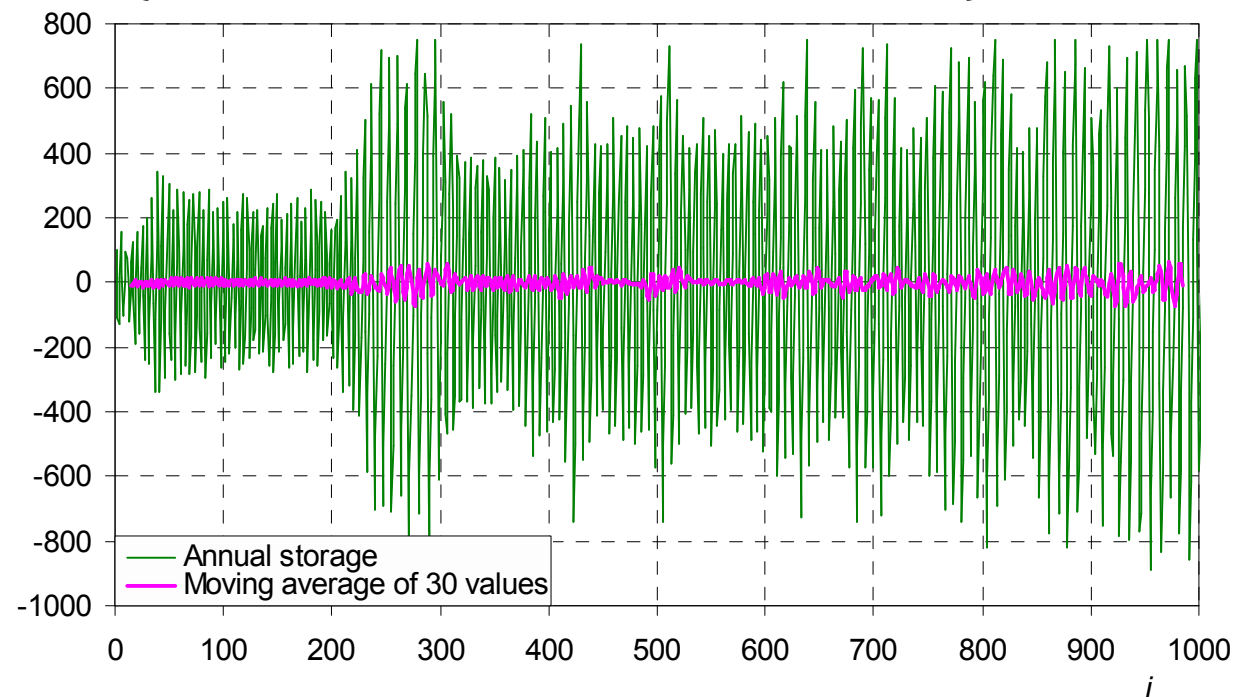
The stochastic representation is good for both short and long horizons, and helps figure out when the deterministic dynamics should be considered or neglected.

A different perspective of long-term predictability and the key consequence of antipersistence

↘ Arguably, when we are interested about a prediction for a long time horizon, we do not demand to know the exact value at a specified time but an average behaviour around that time (the “**climate**” rather than the “**weather**”).

↘ The plot of the soil water for a long period (1000 TU) indicates:

- High variability at a short (annual) scale.
- A flat time average at a 30-year scale (“climate”).
- Peculiar variation patterns.

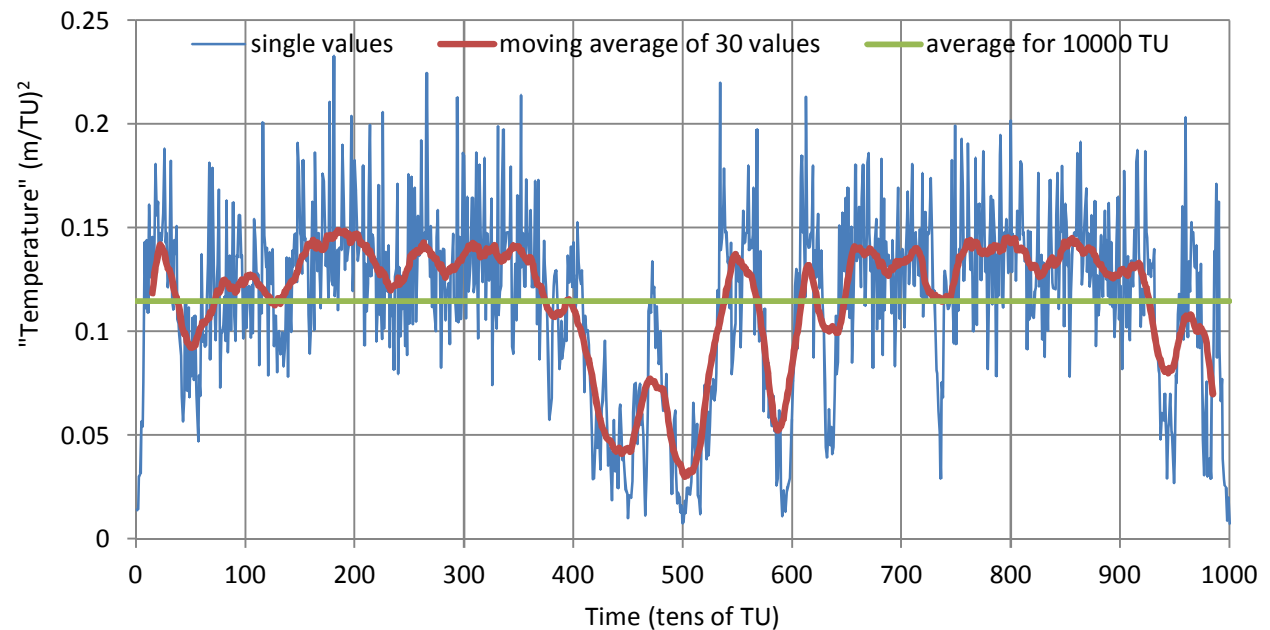


The behaviour quickly flattening the time average is known as **antipersistence** (often confused with periodicity/oscillation, which is an error).

Antipersistence enhances **climatic-type predictability** (prediction of average).

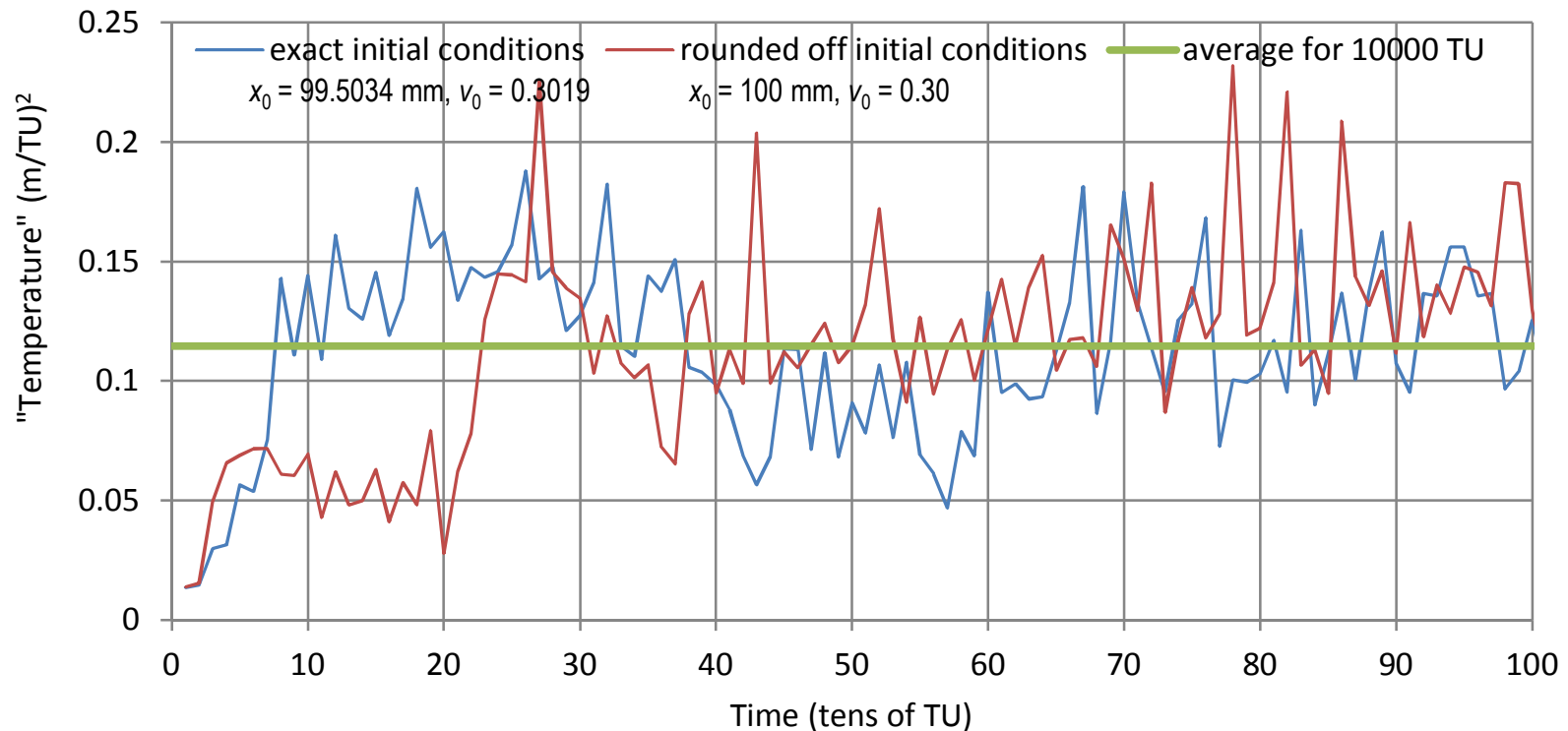
Quantification of variability

- ↘ To study the peculiar variability of the soil water \underline{x}_i , we introduce the random variable $\underline{e}_i := ((\underline{x}_i - \underline{x}_{i-1})/\Delta)^2$, where $\Delta = 1$ TU; \underline{e}_i is an analogue of the “kinetic energy” in the variation of the soil water.
- ↘ Furthermore we introduce a macroscopic variable $\underline{\theta}$, an analogue of “temperature”, which is the average of 10 consecutive \underline{e}_i ; high or low θ indicates high or low rates of variation of soil water.
- ↘ The plot of the time series of θ for a long period (10000 TU) indicates long and persistent excursions of the local average (“the climate”) from the global average (of 10000 values).
- ↘ These remarkable changes are produced by the internal dynamics (no forcing).



The frequent and long excursions of the local average from the global average indicate **long-term persistence**, or **long-term change** (not static conditions). **Persistence/change** are often confused with **nonstationarity**—but this is an error.

Demonstration that variability and persistence entail unpredictability

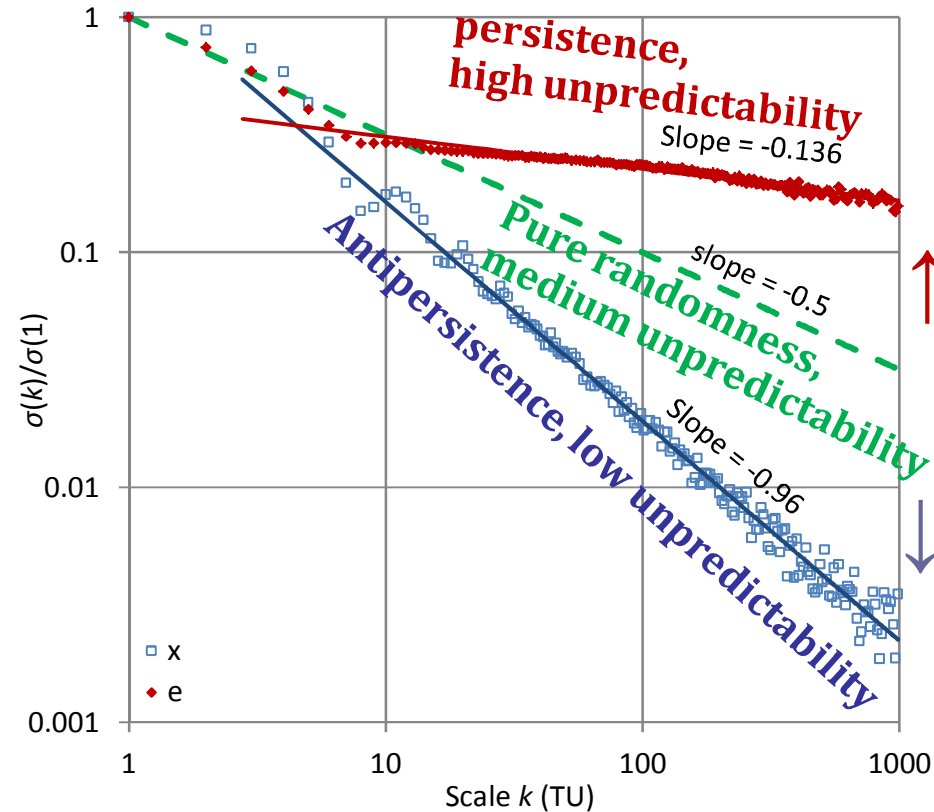


- ↘ The plot shows 100 terms of "temperature" time series produced with exact, as well as rounded off, initial conditions.
- ↘ The departures in the two cases are striking.

Even a **fully deterministic system** is **fully unpredictable** at a climatic time scale when there is **persistence**.

Multi-scale stochastics and predictability

- ↘ For an one-step ahead prediction, a purely random process x_i is the most unpredictable.
- ↘ Dependence and conditioning on observations enhances one-step ahead predictability.
- ↘ However, in the climatic-type predictions, which concern the local average rather than the exact value, the situation is different.
- ↘ The climacogram shown on the right (plot of standard deviation vs. time scale of averaging) shows that in a persistent process (like in e and θ), the uncertainty at long time scales is very high.
- ↘ The reduction due to conditioning on the past is annihilating because of the persistence.



Contrary to the common perception, positive dependence/persistence substantially deteriorates predictability over long time scales—but antipersistent improves it.

From the toy model to real world hydrological systems

WATER RESOURCES RESEARCH, VOL. 48, W09555, doi:10.1029/2011WR011412, 2012

A blueprint for process-based modeling of uncertain hydrological systems

Alberto Montanari¹ and Demetris Koutsoyiannis²

Received 16 September 2011; revised 19 August 2012; accepted 20 August 2012; published 29 September 2012.

[1] We present a probability based theoretical scheme for building process-based models of uncertain hydrological systems, thereby unifying hydrological modeling and uncertainty assessment. Uncertainty for the model output is assessed by estimating the related probability distribution via simulation, thus shifting from one to many applications of the selected hydrological model. Each simulation is performed after stochastically perturbing input data, parameters and model output, this latter population of the model error, whose probability and model parameters. Within this view randomness as an inherent property of hydrological systems, as well as the open research questions. The theoretical real-world and synthetic applications. The relevance of proposing a statistically consistent simulation framework does not require model likelihood computation at all. The results show that uncertainty is satisfactorily captured by the proposed assumptions could be significant in conditions of

expressed by equations (1)–(6) we may see that we passed from the deterministic formulation of the hydrological model expressed by equation (1), i.e., (to replicate it for clarity),

$$Q = S(\Theta, \mathbf{X}) \quad (7)$$

to the stochastic formulation expressed by

$$f_Q(Q) = \int_{\Theta} \int_{\mathbf{X}} f_e(Q - S(\Theta, \mathbf{X}) | \Theta, \mathbf{X}) f_{\Theta}(\Theta) f_{\mathbf{X}}(\mathbf{X}) d\Theta d\mathbf{X} \quad (8)$$

Differences of the “blueprint” with deterministic modelling

■ Deterministic model

$$Q = S(\boldsymbol{\theta}, \mathbf{X})$$

where Q the model output (e.g. river flow), $\boldsymbol{\theta}$ a vector of model parameters and \mathbf{X} a vector of model inputs.

■ Stochastic version—general formulation

$$f_Q(Q) = \int_{\{\boldsymbol{\theta}\}} \int_{\{\mathbf{X}\}} f_e(Q - S(\boldsymbol{\theta}, \mathbf{X}) | \boldsymbol{\theta}, \mathbf{X}) f_{\boldsymbol{\theta}, \mathbf{X}}(\boldsymbol{\theta}, \mathbf{X}) d\boldsymbol{\theta} d\mathbf{X}$$

where $e := Q - S(\boldsymbol{\theta}, \mathbf{X})$ (model error) and f denotes probability density.

■ Simplifying assumptions that enable an easy Monte Carlo implementation:

- Independent parameters and inputs: $f_{\boldsymbol{\theta}, \mathbf{X}}(\boldsymbol{\theta}, \mathbf{X}) = f_{\boldsymbol{\theta}}(\boldsymbol{\theta}) f_{\mathbf{X}}(\mathbf{X})$.
- Representativeness of deterministic model prediction for error conditioning: $f_e(e | \boldsymbol{\theta}, \mathbf{X}) = f_e(e | S(\boldsymbol{\theta}, \mathbf{X}))$.

■ Summary of differences of the stochastic approach with deterministic modelling:

- Models are approximations of reality and model parameters are not physical constants; they are modelled as random variables.
- Precise predictions are infeasible; only probabilities can be calculated.
- One model run is not sufficient; many runs in a Monte Carlo framework can give the required probabilities

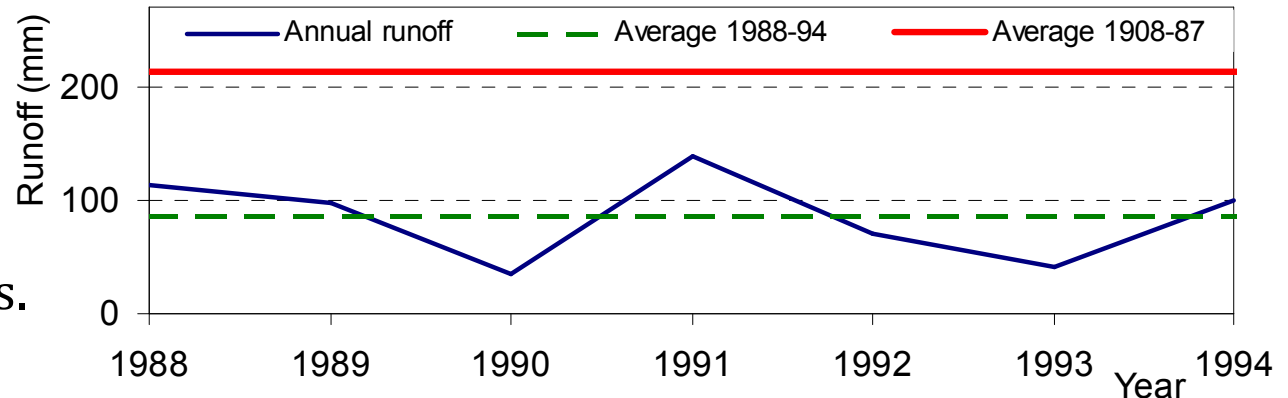
Differences of the “blueprint” with conventional stochastic models

- Conventional stochastic models (e.g. ARMA and even Hurst-Kolmogorov processes) are linear.
- Assuming that the background deterministic model $Q = S(\boldsymbol{\theta}, \mathbf{X})$ is nonlinear (and usually is), the resulting stochastic model for $f_Q(Q)$ will be nonlinear too.
- This nonlinear setting offers a more detailed description of the relevant processes and a more realistic representation of system dynamics at **short lead times**.
- However, at **long lead times** linearity is actually recovered:
 - As entropy approaches its maximum (information is getting lost), linearity reemerges, as it is a consequence of entropy maximization (Koutsoyiannis, 2014b; Efstratiadis et al., 2014).
- Therefore, for long lead times conventional linear stochastic modelling may be preferable as it is simpler and may not be poorer than detailed nonlinear stochastic modelling.

Can we effectively control unpredictable systems?

↘ **Definitely yes**—there is overwhelming engineering experience about it.

↘ An illuminating example is offered by an intense and persistent (lasting 7 years) drought that shocked Athens.



↘ The ingredients for the success story of the Athens drought management include:

- Consistent modelling (see more information in Koutsoyiannis, 2011):
 - ↘ Stochastic hydrological model reproducing long-term persistence.
 - ↘ Advanced decision support tool based on an original and parsimonious stochastic methodology termed **parameterization-simulation-optimization**.
- Construction of new engineering works to improve water resource availability.
- Engagement of the society in water saving practices, which resulted in decrease of the water consumption by 1/3.

Concluding remarks

- ↘ **PP ≠ PUC** (Πάντα ϵ ≠ Prediction Under Change)
- ↘ **PP ≠ PUD** (Πάντα ϵ ≠ Prediction Under Doom)
- ↘ **PP** can help the hydrological community and the society in the following important tasks:
 - ⌊ reconciliation with change;
 - ⌊ reconciliation with uncertainty;
 - ⌊ recognition of the tight connection of change and uncertainty;
 - ⌊ recognition of the inevitability of change and uncertainty;
 - ⌊ recognition of the good sides of change and uncertainty;
 - ⌊ advancement of decision making under uncertainty;
 - ⌊ developing adaptability and resilience for an ever uncertain future;
 - ⌊ promotion of technology and engineering means for planned change to control the environment for the benefit of the society;
 - ⌊ promotion of the importance of honesty in science and its communication to the society;
 - ⌊ advancement of the *Hydrology of Uncertainty*.

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